AN INVESTIGATION OF THE EFFECTS OF POPULATION DYNAMICS ON GROWTH AND TRADE IN AN OVERLAPPING-GENERATIONS GENERAL EQUILIBRIUM MODEL

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ABSTRACT

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In this study, variants of a two-sector, two-factor overlapping-generations model are solved under autarky and free trade scenarios to investigate the effects of population dynamics on growth and trade. Simulation exercises are also performed to develop a deeper understanding of the analytical findings and to visualize the time paths of model variables. These numerical exercises complement analytical solutions, providing significant insights into the nature of initial conditions affecting growth and convergence performance of closed economies. Concerning open economies, possible implications of population growth differentials for the patterns of trade flows between economies that are identical except for population growth rates are explored as in the static Heckscher-Ohlin model. Our analysis shows that population growth rate differentials give way to differences in relative commodity and factor prices, creating the basis for comparative advantages in the same way as suggested by the static Heckscher-Ohlin model. We also show that these demographic differences prevent comparative advantages from getting eliminated in the long-run, thereby allowing trade to continue to occur even after the steady state is reached. Our solutions reveal, however, that trade does not necessarily improve welfare for both parties in the long-run. The explanation we offer for this nicely complements previous studies that obtained similar results using overlapping-generations general equilibrium models within two country set-ups with steady populations.

Keywords: Dynamic trade; Population growth rate; Overlapping-generations general equilibrium model, Heckscher-Ohlin.

ÖZET

NÜFUS DİNAMİKLERİNİN BÜYÜME VE TİCARET ÜZERİNDEKİ ETKİLERİNİN ÇAKIŞAN-NESİLLER GENEL DENGE MODELLEMESİ YOLUYLA İNCELENMESİ

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Bu çalişma, nüfus dinamiklerinin büyüme ve ticaret üzerindeki etkilerini, iki-sektör ve iki-faktörlü bir çakışan-nesiller modelinin çesitli varsayımlar altında elde edilen analitik çözümlerinden çıkan sonuçlar ışığında incelemektedir. Bu analitik çözümler, işaret ettikleri sonuçların daha somut biçimde kavranabilmesi ve model değişkenlerinin zaman içinde izlediği patikaların da izlenebilmesi amacıyla yapılan sayısal simülasyonlarla tamamlanmıstır. Söz konusu sayısal egzersizler, nüfus artış hızları da dahil olmak üzere farklı başlangıç koşullarına sahip kapalı ekonomilerin, büyüme performansları ve daha gelişmiş ekonomileri yakalama potansiyellerinin nasıl farklılaşabileceğine dair çok önemli önseziler sağlamaktadır.

Açık ekonomilere ilişkin olarak ise, nüfusları farklı hızlarda artan ekonomilerin birbirleriyle yaptıkları ticaret kalıplarında zaman içinde gözlenecek değişikler, statik Heckscher-Ohlin modelindeki yaklaşıma paralel biçimde nüfus artış oranları dışındaki tüm karakteristikleri aynı olan ekonomiler göz önüne alınarak incelenmektedir. Analizimiz nüfus artış hızlarındaki farklılıkların her ülkedeki göreli mal ve faktör fiyatlarını Heckscher-Ohlin modelinin önerdiğine benzer biçimde farklılaştıracağını ve bu yolla karşılastırmalı üstünlükler yaratacağını ispatlamaktadır. Sonuçlarımız, yaratılan bu karşılaştırmalı üstünlüklerin nüfus artış hızları farklı kaldığı sürece uzun vadede de korunacağı ve ticaretin durağan-dengede de devam edeceğini de göstermektedir. Öte yandan, elde edilen model çözümleri ticaretin uzun dönem refah artırıcı etkisinin her iki taraf için geçerli olmayabileceğini de ortaya koymaktadır. Bu ilginç gözleme ilişkin olarak sunduğumuz açıklama, iki ülke arasındaki dinamik ticaret dengesini çakışan-nesiller çerccevesi bağlamında ancak nüfus artışına izin vermeksizin ele alan daha önceki çalışmalardan oluşan literatürü tamamlayıcı niteliktedir.

Anahtar sözcükler: Uluslararası ticaretin dinamik dengesi; Nüfus artış hızı; Çakışan-nesiller genel denge modeli; Heckscher-Ohlin modeli.

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CHAPTER I

INTRODUCTION

Globally observed decline in fertility and mortality rates gradually lower population growth rates, eventually causing a visible increase in the share of elderly population around the world. In countries where this demographic process has worked faster than the others, the population age pyramid's base already began to shrink, while its summit started to widen. Most members of the OECD, for instance, have been witnessing a rapid reduction in the population growth rates and an acceleration in the pace of aging. This process is projected to continue until the overall dependency rate (i.e., the ratio of population outside the working age to the working age population) exceeds 70% in the 2040s (Kenç and Sayan (2001)). Increases in this ratio are likely to have significant implications for the OECD economies, as well as the countries with which they have strong ties, since the movement of factors and commodities across borders serves as a channel transmitting the effects of demographic changes in one country onto other economies. In their pioneering work, Kenç and Sayan (2001) showed that these demographic spill over effects could be important. They noted, based on their results, that small economies trading commodities and capital with large economies may be exposed to the effects of population aging earlier than their own demographic transitions would have implied.

Many developing countries are yet to face a similar decline in the population growth rate indeed, and hence, still have populations that are predominantly young. The existing differences in the population growth rates and hence variations in age profiles of populations in different parts of the world are not likely to be eliminated for several decades to come. In fact, if this disparity in the population growth rates between the developed and the developing parts of the world is to prevail as projected by demographers, not only the labor forces will continue to diverge, but also variations in the age profiles of populations will become increasingly visible. This differential speed of population aging in the developing and developed areas will necessarily affect the relative abundance of capital as well. The labor supply will eventually begin to fall wherever population aging sets in and capital formation will be slowed down by the associated decrease in savings. By the modern theory of international trade, differences in relative factor endowments form the theoretical basis for differences in commodity and factor prices underlying trade and factor movements. In other words, differences in population growth rates can be viewed as a potentially major determinant of commodity and factor flows across borders.

The main objective of this dissertation is to study growth and trade implications of population dynamics within a dynamic general equilibrium framework.

In order to use a dynamic structure that allows for the age composition of populations to differ across countries later on, the overlapping-generations framework was found to be suitable for modeling our prototype closed economy. Moreover, analysis of the direction and magnitude of changes in trade flows in response to changes in factor endowments requires that at least two commodities and two factors of production be considered. Thus, a two-commodity, two-factor, two-generation framework was chosen for our prototype economy. The changes in relative factor endowments resulting from changes in age composition of population over time were captured through the addition of population dynamics.

A simple version of the two-sector overlapping-generations economy in Galor (1992*b*) was considered to model each economy's autarky equilibrium. Economic activity in this model extends over infinite discrete time and is conducted under perfect competition and certainty. The consumption side of this economy consists of agents living in a typical overlapping-generations world. They live for two periods and have perfect foresight. Agents are homogeneous both inter-generationally and intra-generationally. At any given period, two types of individuals are alive: young that are born in the current period, and are living the first period of their lives, and olds who were born in the previous period and are living the last period of their lives. In their first period of life, agents work by inelastically supplying their labor endowment, earn the competitive market wage, and decide on how much to consume and how much to save. In the second period of life, agents just rent their savings and consume all their wealth.

The production side consists of two sectors: Two goods are produced according to constant returns to scale Cobb-Douglas production technologies. However, unlike the standard practice used in two-sector models in the literature, such as Galor (1992*b*) and Azariadis (1993), the non-perishable good serves as an investment as well as a consumption good, whereas the perishable good serves as a consumption good only. The production environment is competitive and labor and capital are perfectly mobile across sectors.

Following a survey of the existing literature in Chapter 2, the discussion of this dissertation begins with an in-depth analysis of the long-run closed-form solutions for this version of the closed economy model. Analytical solutions for such an economy are shown to be feasible to obtain for the steady state values of all variables. The discussion in this chapter shows that the dynamics of such an economy can be expressed through a single non-linear difference equation in terms of one variable only as in Galor (1992*b*).

Chapter 4 presents sample numerical solutions to develop a better understanding of the dynamics of the economy through a phase diagram analysis. With the help of these simulation exercises, not only do we provide a clear idea about how the economy moves from its initial endowments to the long-run equilibrium, but we also provide a visualization of the model variables' time paths that are analytically challenging to derive.

In Chapter 5, the long-run closed-form solutions of the autarkic economy obtained in Chapter 3 are used to explore the effect of population growth rates on the economy's steady state key variables. This chapter describes the role that differences in population growth rates across nations could play as a determinant of long-run comparative advantages. Unequal population growth rates give rise to differentials in wage rates and rentals for capital under autarky conditions causing costs of production and relative prices to differ, hence creating the grounds for trade. Simulation experiments are performed again to help visualize the time paths of the economy's key variables, complementing the analytical findings.

In Chapter 6, the closed economy model is extended to allow for trade to see the effects of population growth rate differentials within a dynamic Heckscher-Ohlin framework. Two countries, similar in every aspect except for the population growth rates, are allowed to trade using the 2x2x2x2 extension of autarky model developed for this purpose. Consistently with the predictions of the static Heckscher-Ohlin model, the addition of dynamics does not affect the direction of trade between the two countries, but trade is shown not to necessarily lead to welfare gains for both countries. That trade might not be Pareto-superior to autarky is consistent with previously obtained results by Sayan and Uyar (2001), Sayan (2002) and Sayan (2005) based on numerical solutions of the trade model with growing populations and for different ranges of parameters. This result is also consistent with a number of studies including Mountford (1998) where it is demonstrated that if a dynamic two period overlapping-generations structure is added to the standard Heckscher-Ohlin model under stationary populations, then the static implications of international trade can be reversed over time.

Chapter 7 concludes the dissertation. The lessons that can be derived from the study and the contributions to the existing literature can be summarized as follows:

First, it is shown that the long-run closed-form solutions of a 2x2x2 overlapping-generations autarky economy are feasible to obtain when one good is allowed to be used for consumption as well as investment purposes with the other serving as a consumption good only.

Second, it is demonstrated that difference in population growth rates across nations give way to differences in relative commodity and factor prices, creating the basis for comparative advantages and hence determining the pattern of trade between nations in the same way as suggested by the static Heckscher-Ohlin model.

Third, the long-run closed-form solutions for a 2x2x2x2 overlapping-generations world economy are derived and used to analyze the nature of dynamic free trade equilibrium.

Fourth, it is established that population growth rate differential prevent comparative advantages from getting eliminated in the long-run despite the equalization of prices, thereby allowing trade to continue to occur even after the steady state is reached.

Fifth, the welfare levels under autarky and trade are compared and it is shown that the static Heckscher-Ohlin results can not be generalized to hold in a dynamic setting like the one considered, since, trade does not necessarily improve welfare for both parties in the long-run. This is because the high population growth country will behave as a large country capable of setting the terms of trade in the long-run, as a result of the parallel growth in its share of total world output and population. For example, China currently having one-fifth of the world's population with a relatively high population growth rate[1] has been increasingly integrating to the world trading system. Stronger integration of China to the global economy has already started affecting world prices of many commodities. While an analysis of the impact of China's integration to the global markets is beyond the scope of this dissertation, it serves as a good example illustrating the relevance of some of the issues tackled here.

Last but not least, all the findings above are supported and steady state solutions are complemented by simulation exercises that visually describe the time paths of all model variables under study.

[1] China's average annual population growth rate, between 1980 and 2002 is 1.2%, whereas that for the high income countries is 0.7%,WDI (2004).

CHAPTER II

LITERATURE REVIEW

Several studies in the literature noted and studied the effects that the population growth rate can have on trade patterns and growth performance of national economies. These studies can be classified according to the type and structures of the models used. First, there is the North-South trade literature. Second are the studies incorporating differences in population growth rates into dynamic growth models to explore implications of these differences for trade. Third group of studies are based on overlapping-generations general equilibrium models.

Motivated by the observation that population growth rates do differ between the Northern and Southern hemispheres of the globe, several researchers attempted to investigate the consequences of this existing gap. Matsuyama (2000) looks at the role that population size and technology can play in trade within a Ricardian framework which explains comparative advantages by differences in technology. The main contribution of this study is to replace the standard homotheticity property of consumption and employ in lieu a continuum of goods. Goods at the lower end of the spectrum are consumed by all households. As their income levels go up, the households expand their range of consumption by adding higher-indexed goods to their baskets. In order to explore the implications of trade, two countries with nonhomothetic preferences are considered: The North is developed with high income, has a comparative advantage and thus specializes in the production of higher-indexed goods (goods with high elasticities of demand). The South is underdeveloped with low income, has a comparative advantage and thus specializes in the production of lower-indexed goods (goods whose demand has low income elasticity). Within this framework, the faster population growth in the South can generate product cycle phenomena. It is argued that South experiences a secular decline in its terms of trade, and the lower-end industries in North move continuously to the South. As the prices of imports from South declines, the northern households expand their range of consumption continuously towards higher indexed goods, thereby giving birth to new industries in North. Another implication of this asymmetry of demand complementarities between goods is that as a result of faster population growth and the uniform productivity growth in the South associated with an improvement in global productivity, the welfare gain of productivity growth is unevenly distributed. North can capture all the benefits of its own uniform productivity growth, whereas South may lose from its own uniform productivity growth. When the price of lower-indexed goods decline, demand for higher-indexed goods will increase as the households respond to the higher real income resulting from the reduction in prices of lower-indexed goods by adding higher-indexed goods to their consumption baskets.

One interesting question in the context of North-South trade relations is whether the South would catch up to the North in standard of living if the South share the same standard of living, given that the North starts with more capital stock in per capita terms than the South. Chen (1992) shows that the world economy will approach a long-run equilibrium where the rich countries remain rich and the poor countries remain poor.

Next are studies including the population growth rate into dynamic growth models of open economies trading with others. The standard Heckscher-Ohlin (H-O) model of international trade has been employed to analyze the long-run equilibria of open economies and one major issue studied has been the determinants of comparative advantages in the long-run (Oniki and Uzawa (1965), Findlay (1970)). Oniki and Uzawa (1965) and Bardhan (1965) extended the two-sector growth model to a two-country world, demonstrating that in a world in which the propensities to save differ across countries, the country with the higher propensity to save exports the capital intensive good in the long-run. In their seminal work where they used a dynamic two-country, two-commodity, two-factor growth model, Oniki and Uzawa (1965) allowed for differences in population growth rates in order to investigate the effects of capital accumulation and labor force growth on international equilibrium over time. It is found that given the technological knowledge and the tastes of consumers in both countries, the volume and terms of trade and the pattern of specialization depend upon the quantities of productive factors endowed in both countries. Findlay (1970) extended the Oniki and Uzawa model by adding a non-traded capital good and established the relationship between trade patterns of a small three-sector economy, and saving propensities and rates of population growth. He found that when international capital movement is not allowed, the long-run pattern of comparative advantage depends ultimately on the propensity to save and the growth rate of labor force. Stiglitz (1970) demonstrated, in a two-country two-sector infinite horizon world where the rates of time preference differ across countries, that factor price equalization would not hold in the long-run. Matsuyama (1988) considered the trade patterns of a small three-sector economy in a life cycle model.

It appears to be the general consensus of this literature that the main determinant of long-run comparative advantage is the countries' saving rates. The models that do endogenize the saving rates attributes the difference in saving rates and hence long-run comparative advantage to a difference in preferences; in particular, a difference in agents' time discount factors among countries.

Yet, explaining trade in terms of such differences in preferences is not in the spirit of the Heckscher-Ohlin model which suggests that trade arises mainly because of differences in relative factor endowments rather than differences in preferences or production technologies. One well known result of the existing neoclassical growth models is that in the long-run all countries with identical preferences will always converge to the same steady state, independent of initial conditions, failing to explain enormous differences we observe in per capita income levels across countries in the real world. Chen (1992) demonstrates by employing a two-country two-good, two-factor growth model that once international trade is incorporated into a neoclassical growth model, countries with different initial per capita income levels will no longer converge to the same steady state. The difference in initial income levels across countries persist in the long-run. Hence, while trade may still be associated with a difference in saving rates among countries, this difference is not caused by a difference in preferences. Rather, it is caused by a difference in initial factor proportions.

While growth models with at least two commodities and two factors of production serve well for the analysis of the direction and magnitude of changes in trade flows in response to changes in factor endowments, they can not show the effects of changes in the age composition of population on relative endowments (Sayan (2002)). Proper modeling of the effects of changes in age profile of population on a wide range of variables such as growth, trade, sectoral adjustments, etc., calls for multi-sector, overlapping-generations models.

Third group of studies are based on overlapping-generations general equilibrium models that can address the effects of the changes in age composition of population on relative endowments. Fried (1980) used a simple overlapping-generations model with two commodities, two generations, and one fixed factor of production and compared steady state solutions under free trade and autarky by assuming zero population growth rate. He showed that free trade may make at least one country worse off relative to autarky under certain conditions. An innovation that could make everyone in the current generation better off now and in the future, Fried argued, may worsen the level of welfare for all agents born after the innovation despite the fact that they have the same tastes and the same life-cycle endowments as those agents who instituted the innovation. In other words, if individuals have finite lives, then the gains associated with a move to a market determined Pareto-efficient equilibrium may only be transitory, accruing entirely to some of those alive at the time of innovation. Fried (1980) showed that international trade increases the value of the consumption good, and the change in relative output prices from their autarkic levels to those prevailing in the rest of the world causes factor prices to change. If the factor price effect reduces the real wage and the change in the value of the consumption good is not too large, future generations may all lose from the country moving to free trade in consumption goods.

Buiter (1981) used a deterministic model of two countries, each one of which behaves as in a Samuelson-Diamond type of overlapping-generations model and produces one identical good that can be used as a consumption or a capital good in order to explain international capital movements based on differences in time preferences. He evaluated the short-run and long-run welfare implications of a change from a situation of trade and financial autarky to one of openness in trade and finance. He showed that the country with a higher pure rate of time preference (whose residents consume more in the first period of their lives at given wage rate and interest rate) has a steady state current account deficit if the population growth is positive, whereas the low-time preference country runs a current account surplus in the steady state but not necessarily outside it. Concerning welfare, he found that the ranking of stationary utility levels under autarky and openness is ambiguous.

Galor (1992*b*) was the first to formally develop the two-sector overlapping-generations model as a counterpart to the two-sector growth model employed for instance, by Benhabib and Nishimura (1985) and in earlier studies by Oniki and Uzawa (1965). The model that Galor (1992*b*) studied is an extension of the one-sector overlapping-generations model of Diamond (1965), where two goods are produced: a perishable consumption good and an investment good. Azariadis (1993) claims that distinguishing of investment goods from consumption goods in growth theory permits the price of capital to deviate from its cost of reproduction which is not the case in the standard one-sector overlapping-generations model. This deviation provides another reason for studying multi-sector models. In the single dimensional dynamic system (with a single initial condition) that characterizes the single-sector overlapping-generations production economy, gross substitution (individuals' saving is an increasing function of the real return to capital) ensures the global determinacy of perfect-foresight equilibrium (eg., Galor and Ryder (1989)). However, in the multi-dimensional dynamic system that characterizes multi-sector models, gross substitution in consumption is not sufficient to rule out indeterminacy. It is shown by Galor (1992*b*) that a perfect-foresight two sector overlapping-generations model has a globally unique equilibrium if the following three conditions hold:

1. gross substitutability in first and second period consumption,
2. the investment good is capital intensive, and
3. second period consumption is a normal good.

Several attempts based on Galor (1992*b*) study have been employed to provide dynamic foundations for various characteristics of international economics.

Galor and Lin (1994) employed a two-sector overlapping-generations along the lines of Galor (1992*b*), which follows the traditional two-sector growth model where the economy is characterized by a consumption good sector and an investment good sector. Within this framework, they derived the changes in the world relative prices and factor prices that result from shocks to technology explicitly. Based on these fundamental relationships, the current account (domestic savings minus domestic investment) of a small open economy which specializes in the production of the investment good was characterized in response to the a deterioration in terms of trade that is trigged by a technological shock in the world economy. They found that factor intensities in the production sectors as well as the nature of the shock are significant in the determination of the response of the current account to a deterioration in the terms of trade.

Mountford (1998) utilized the Galor (1992*b*) model and showed that the static implications of international trade in a two-country, two-sector, two-factor Heckscher-Ohlin world economy model can be reversed over time if a dynamic two period overlapping-generations structure is added to this model. Moreover, he showed that international trade between two countries similar in every respect except for time preferences, hence saving rates, can cause conditional convergence and can reduce the steady state welfare in one economy without increasing the welfare in the other.

Galor and Lin (1997) established dynamic microfoundations for the fundamental proposition of the most influential model of international trade theory, the Heckscher-Ohlin model. They looked at a two-country, two-sector overlapping-generations world where countries differ in their rate of time preferences, using a model along the lines of the traditional two-sector growth model (e.g., Uzawa

(1964), Srinivasan (1964), Oniki and Uzawa (1965), and Shell (1967)), and two-sector overlapping-generations model (Galor (1992*b*)). Buiter (1981) established dynamic foundations for the patterns of international lending and borrowing, within a framework of two Diamond-type overlapping-generations economies which differ in their rates of time preferences. Galor (1986) established dynamic foundations for the patterns of international labor migration within the same framework. Eaton (1987) provided the specifications for the specific factor model.

In contrast to Findlay (1970) and Matsuyama (1988), Galor and Lin (1997) considered large countries, permitting a comprehensive general equilibrium analysis in which the terms of trade dynamics are endogenously determined. They demonstrated that in a two-country two-sector overlapping-generations world in which countries differ in their rates of time preference and the investment good is capital intensive, the higher the rate of time preference, the lower the steady state level of the capital-labor ratio and the lower the steady state relative price of the capital intensive good.

Guilló (2001) employed the Galor model in order to explore the type of the relationship between the trade balance and the terms of trade. She showed that in most cases, the relationship between the trade balance and the terms of trade is positive, and offered explanations concerning the negative relationship between the terms of trade and the trade balance that can arise in large countries.

The growth literature suggests that the population growth rate is one of the main determinants of long-run comparative advantages and a major factor that establishes the pattern of trade for an open economy, Oniki and Uzawa (1965), Findlay (1970). However, the overlapping-generations literature has largely overlooked the implications of population dynamics for trade focusing on other issues (such as current account of a small open economy, Galor and Lin (1994); differences in time preferences, Galor and Lin (1997)) instead, by assuming zero population growth rate.

Only very recently, Sayan and Uyar (2001) and Sayan (2002) noted the significance of population dynamics and showed, using numerical solutions from a 2x2x2 OLG model, that exogenously given and distinct population growth rates may create incentives for trade but trade may not generate welfare gains for both parties. Sayan (2005) complemented these results by showing that the same conclusions would essentially hold, even when population growth rates decline gradually over time.

In parallel with the endogenous growth literature, the neoclassical growth models with diverging populations raised the possibility that countries may grow without bounds in terms of per capita income and they may do so at different rates. This means that international inequality of per capita incomes will not only exist but will also get worse over time. The presence of this differences in population growth rates across countries can give rise to this international inequality as suggested by, Deardorff (1999). With diverging populations, the country with the largest population growth rate comes to dominate the world population, in the sense that its share of world population goes to one regardless of how small it may have started. It is for this reason that other contributors to the literature on economic growth have tended to ignore the case of diverging populations, dismissing it as converging to a single closed economy. This calls for a more careful care, Deardorff (1994). The possibility that population growth rates differ across countries has been neglected in the literature, not because it is unlikely to arise in the real world, but because it has been considered as uninteresting.

Galor and Weil (2000) analyzed the historical evolution of the relationship between population growth, technological change and the standard of living. They characterized the process of economic development by three distinct regimes:

The Malthusian Regime, the Post-Malthusian Regime and the Modern Growth Regime. In the Malthusian regime technological progress and population growth were glacial by modern standards and income per capita was roughly constant. There existed a positive relationship between income per capita and population growth rate. During the post-Malthusian regime, income per capita grew although not as rapidly as it would during the modern growth regime and there existed a Malthusian (positive) relationship between income per capita and population growth rate. The modern growth regime is characterized by a steady growth in both income per capita and the level of technology. There is a negative relationship between the level of output and the growth rate of population. The highest rates of population growth are found in the poorest countries and many rich countries have population growth rates near zero. The historical evidence suggests that the key event that separates the Malthusian and the post-Malthusian regime is the acceleration in the pace of technological progress, whereas the event that separates the post-Malthusian and the modern growth eras is the demographic transition that followed the industrial revolution. Majority of the studies in the existing literature have been oriented towards the modern regime trying to explain the negative relation between income and population growth.

In their 2003 study, Galor and Mountford combined the elements of endogenous growth models with overlapping-generations, general equilibrium models. They considered an overlapping-generations economy where two goods are produced using up to three factors of production: Skilled labor, unskilled labor and land. In each of the sectors of the economy production may take place with either an old technology or a new one. Individuals live for two periods and get utility from consumption of the agricultural good, consumption of the manufactured good and the total potential income from offsprings. It is suggested that international trade has an asymmetrical effect on the evolution of industrial and non-industrial economies. While in the industrial nations the gains from trade were directed primarily towards investment in education and growth in output per capita, a significant portion of the gains from trade in non-industrial nations was channeled towards population growth.

Galor and Mountford (2003) argue that the rapid expansion of international trade in the second phase of the industrial revolution has played a major role in the timing of demographic transitions across countries and has therefore been a significant determinant of the distribution of world population and a prime cause of the divergence in income levels across countries in the last two centuries. The argument goes as follows: In the second phase of the industrial revolution, international trade enhanced the specialization of industrial economies in the production of industrial, skilled intensive goods. The associated rise in the demand for skilled labor has induced a gradual investment in the quality of the population in industrial economies, expediting a demographic transition, stimulating technological progress and further enhancing the comparative advantage of these economies in the production of skilled intensive goods. In the non-industrial economies, international trade has generated an incentive to specialize in the production of unskilled intensive, non-industrial goods. The absence of significant demand for human capital has provided limited incentives to invest in the quality of the population and the gains from trade have been utilized primarily for a further increase in the size of the population, rather than the income of the existing population. The demographic transition in these non-industrial economies has been significantly delayed, further increasing their relative abundance of unskilled labor, enhancing their comparative disadvantage in the production of skilled intensive goods and delaying the process of development. The authors suggested that sustained differences in income per capita and population growth across countries may be attributed to the contrasting role that international trade had on industrial and non-industrial nations.

In this study, we consider the standard set-up of the static Heckscher-Ohlin (H-O) model to examine the implications of the long-run effect of population differentials on trade. As described in Salvatore (2001), the static H-O framework, in its standard form is characterized by the following assumptions:

1. There are two countries, two commodities and two factors of production.
2. Both countries use the same technology in production.
3. One commodity is capital-intensive and the other is labor-intensive. More precisely, the capital-labor ratio (K/L) is higher for the capital-intensive commodity than for the labor-intensive commodity.
4. Both commodities are produced under constant returns to scale in both countries.
5. There is incomplete specialization in production in both counties. That is, even with free trade both countries continue to produce both commodities.
6. Demand preferences are identical in both countries.
7. There is perfect competition in both commodities and factor markets in both countries.
8. There is perfect factor mobility within each country but no international factor movements.
9. There are no transportation costs, tariffs, or other obstructions to the free flow of international trade.
10. All resources are fully employed in both countries.
11. International trade between the two countries is balanced.

The Heckscher-Ohlin theorem states that a country will export the commodity whose production requires the intensive use of the country's relatively abundant and cheap factor and import the commodity whose production requires the intensive use of the country's relatively scarce and expensive factor. Hence, the H-O framework isolates the difference in relative factor abundances (or factor endowments) as the basic determinant of comparative advantage and international trade. More precisely, the difference in relative factor abundances and prices is the cause of the pre-trade differences in relative commodity prices between two countries.

Within this static set-up, relative factor endowments of countries are different and do not change over time. We extended this static set-up to a dynamic one by imposing an overlapping-generations structure to the model under non-stationary populations so as to allow factor supplies to be determined within the model itself. This results in a generation of a replica of the static H-O framework at each period, while considering the evolution of the main factors of production through differential population dynamics that we introduced. In other words, inequality of population growth serves as the driving force behind the changes in relative factor endowments at each period. In fact, differences in population growth rates not only affect the growth of labor supply but also that of capital stock through the savings of the young.

According to Salvatore (2001), H-O model is useful in explaining international trade in raw materials, agricultural products, and labor-intensive manufacturers, which is a large component of the trade between developing and developed countries.

The results in the present study link up well with the discussion in Galor and Mountford (2003), where the authors suggest that the observed variation in the speed of demographic transition between industrial and non-industrial nations can be explained with the historically observed differences in the way they have distributed their respective gains from trade. In their model, Galor and Mountford consider endogenous fertility and technological change, and distinguish between unskilled and skilled labor whose paths, they argue, have differed across industrial and non-industrial nations due to the use by the former of gains from trade in education, and hence, in improving skill levels of labor. The present study, on the other hand, assumes away technological change and human capital formation (and hence different skill levels of labor) to highlight the effects of exogenous differences in population growth on trade patterns within a set-up that is completely H-O in spirit.[2] While their purposes and hence the model assumptions employed in the two studies differ, our finding that the differences in relative endowments of capital and labor induced by the differing speeds of demographic transition alone will not be sufficient to render trade mutually beneficial in the long-run is not in contradiction with the arguments in Galor and Mountford.

[2]Deardorff (1999) addresses a related problem to Galor and Mountford (2003) by studying the effect of diverging population growth rates on worldwide distribution of income based on exogenous population growth rates as in the present study.

CHAPTER III

THE BASIC MODEL

The model used is an infinite horizon two-period overlapping-generations model with perfect foresight. In this model, one young (y) and one old (o) generation exist at any point in time. Individuals in this overlapping-generations economy work when young and are retired when old. The young decide on current consumption and anticipated old age consumption based on their preferences and lifetime resources. Preferences of an individual living in this economy are of the Cobb-Douglas type. The lifetime resources of young consist of wage income only, whereas those of old consist of savings accumulated when young plus income earned on their savings. No bequests nor any net intergenerational transfers are allowed in this model. That is, old spend all their income on consumption.

On the supply side, two commodities are produced in a competitive environment, by using labor and capital under constant returns to scale Cobb-Douglas type production technologies that are different across commodities. Labor is supplied by the current young, whereas capital is supplied by the current elderly, corresponding to the savings of the last period's young generation.

As differently from overlapping-generations general equilibrium models in such studies as Galor (1992*b*) and Azariadis (1993), our model allows good 1 to be used for consumption as well as investment purposes. While this makes the model relatively more realistic, it also adds to the complexity of the utility maximization problem, since the consumers are now required to decide how much to consume of each good every period.

3.1 Consumption and Saving

3.1.1 Utility Maximization Problem

At every period t, a generation made up of N_t individuals is born. Population grows at the rate n so that $N_t = (1+n)N_{t-1}$. Individuals live for two periods. They work in the first period and retire in the second period. Individuals born at time t are characterized by their intertemporal utility function $u(c_{1yt}, c_{2yt}, c_{1ot+1}, c_{2ot+1})$ defined over nonnegative consumption bundles during the first and the second periods of their lives. The individual's utility is of the Cobb-Douglas similarly to Auerbach and Kotlikoff (1987) where a one-commodity Cobb-Douglas utility is used, and to Sayan and Uyar (2001) and Sayan (2005) where a two-commodity Cobb-Douglas utility is employed.

$$u(c_{1yt}, c_{2yt}, c_{1ot+1}, c_{2ot+1}) = (c_{1yt}^{\theta} c_{2yt}^{1-\theta})^{\mu} (c_{1ot+1}^{\theta} c_{2ot+1}^{1-\theta})^{1-\mu}, \tag{3.1}$$

where $0 < \theta < 1$ and $0 < \mu < 1$. For all periods t, individuals born and living the first period of their lives at time t inelastically supply a fixed amount of labor, $\bar{l}$; earn labor income at the competitive wage rate, w_t, and decide on how to allocate it between first period consumption of good 1 and 2 (c_{1yt},c_{2yt}), and savings, s_t. Given the price, p_t, of the consumption good (good 2) in terms of the investment-consumption good (good 1) at time t,

$$s_t = w_t\bar{l} - (c_{1yt} + p_t c_{2yt}). \tag{3.2}$$

Individuals save by purchasing the investment-consumption good (good 1) which is the only store of value in the economy. Savings bring interest earnings at the rate of r_{t+1} the next period. In the second period, the individual retires and consumes c_{1ot+1} units of good 1, and c_{2ot+1} units of good 2 by spending all his capital income from previous period's savings. Hence, the second period consumption of an individual born at time t is:

$$c_{1ot+1} + p_{t+1}c_{2ot+1} = (1 + r_{t+1})s_t. \tag{3.3}$$

Plugging the expression of s_t from on (3.2) into (3.3) and arranging terms yields the budget constraint:

$$c_{1yt} + p_t c_{2yt} + \frac{1}{1 + r_{t+1}}(c_{1ot+1} + p_{t+1}c_{2ot+1}) = w_t\bar{l}. \tag{3.4}$$

This condition states that the present value of the individual's life time consumption equals his initial wealth (which is zero) plus the present value of life time labor income (which is $w_t\bar{l}$). Hence, the individual's problem can be formulated as follows:

$$\max \qquad (c_{1yt}^{\theta}c_{2yt}^{1-\theta})^{\mu}(c_{1ot+1}^{\theta}c_{2ot+1}^{1-\theta})^{1-\mu}$$

subject to

$$\begin{aligned} c_{1yt} + p_t c_{2yt} + \frac{1}{1 + r_{t+1}}(c_{1ot+1} + p_{t+1}c_{2ot+1}) &= w_t\bar{l} \\ c_{1yt}, c_{2yt}, c_{1ot+1}, c_{2ot+1} &\geq 0 \end{aligned}$$

Now, we can write down the following Lagrangian for the individual's problem.

$$\begin{aligned} L &= (c_{1yt}^{\theta} c_{2yt}^{1-\theta})^{\mu} (c_{1ot+1}^{\theta} c_{2ot+1}^{1-\theta})^{1-\mu} \\ &+ \lambda \left[w_t \bar{l} - (c_{1yt} + p_t c_{2yt} + \frac{1}{1+r_{t+1}} (c_{1ot+1} + p_{t+1} c_{2ot+1})) \right], \end{aligned} \tag{3.5}$$

where λ is the marginal utility of consumption which is positive.

The first order conditions are:

$$\mu\theta c_{1yt}^{\mu\theta-1} c_{2yt}^{\mu(1-\theta)} c_{1ot+1}^{\theta(1-\mu)} c_{2ot+1}^{(1-\theta)(1-\mu)} = \lambda, \tag{3.6}$$

$$\mu(1-\theta) c_{2yt}^{\mu(1-\theta)-1} c_{1yt}^{\mu\theta} c_{1ot+1}^{\theta(1-\mu)} c_{2ot+1}^{(1-\theta)(1-\mu)} = \lambda p_t, \tag{3.7}$$

$$\theta(1-\mu) c_{1ot+1}^{\theta(1-\mu)-1} c_{1yt}^{\mu\theta} c_{2yt}^{\mu(1-\theta)} c_{2ot+1}^{(1-\theta)(1-\mu)} = \frac{\lambda}{1+r_{t+1}}, \tag{3.8}$$

$$(1-\theta)(1-\mu) c_{2ot+1}^{(1-\theta)(1-\mu)-1} c_{1yt}^{\mu\theta} c_{2yt}^{\mu(1-\theta)} c_{1ot+1}^{\theta(1-\mu)} = \frac{\lambda p_{t+1}}{1+r_{t+1}}. \tag{3.9}$$

Substituting (3.6) into (3.7) yields

$$c_{2yt} = \left(\frac{1-\theta}{\theta}\right) \left(\frac{1}{p_t}\right) c_{1yt}. \tag{3.10}$$

Substituting (3.8) into (3.9) yields

$$c_{2ot+1} = \left(\frac{1-\theta}{\theta}\right) \left(\frac{1}{p_{t+1}}\right) c_{1ot+1}. \tag{3.11}$$

Substituting (3.6) into (3.8) yields

$$c_{1ot+1} = \left(\frac{1-\mu}{\mu}\right)(1+r_{t+1})c_{1yt}. \tag{3.12}$$

Substituting (3.7) into (3.9) yields

$$c_{2ot+1} = \left(\frac{1-\mu}{\mu}\right)\left(\frac{p_t}{p_{t+1}}\right)(1+r_{t+1})c_{2yt}. \tag{3.13}$$

Hence, writing all consumption variables in terms of first period consumption of good 1, c_{1yt} requires finding c_{2ot+1} in terms of c_{1yt}. Substituting (3.12) into (3.11) yields

$$c_{2ot+1} = \left(\frac{1-\mu}{\mu}\right)\left(\frac{1-\theta}{\theta}\right)\left(\frac{1}{p_{t+1}}\right)(1+r_{t+1})c_{1yt}. \tag{3.14}$$

Substituting (3.10), (3.12), (3.14) into the budget constraint, (3.4), yields

$$c_{1yt} = \mu\theta w_t\bar{l}. \tag{3.15}$$

Substituting (3.15) into (3.10) yields

$$c_{2yt} = \mu(1-\theta)\frac{w_t\bar{l}}{p_t}. \tag{3.16}$$

Substituting (3.15) into (3.12) yields

$$c_{1ot+1} = (1-\mu)\theta(1+r_{t+1})w_t\bar{l}. \tag{3.17}$$

Substituting (3.15) into (3.14) yields

$$c_{2ot+1} = (1-\mu)(1-\theta)(1+r_{t+1})\frac{w_t\bar{l}}{p_{t+1}}. \tag{3.18}$$

An examination of (3.15) through (3.18) reveals that the ratio of the optimal amount of one good to the other is independent of the level of income at any given price ratio. For instance, during the first period, the ratio of good 1's consumption to that of good's 2 is

$$\frac{c_{1yt}}{c_{2yt}} = \left(\frac{\theta}{1-\theta}\right) p_t. \tag{3.19}$$

Similarly, the second period consumption proportion of both goods is

$$\frac{c_{1ot+1}}{c_{2ot+1}} = \left(\frac{\theta}{1-\theta}\right) p_{t+1}. \tag{3.20}$$

The same thing can also be shown to hold for consumption ratios across periods. Hence, the demand pattern in this model is *homothetic*, due to the Cobb-Douglas type preferences.

3.1.2 Parameters of the Utility Function

Examining (3.19) and (3.20), we notice that the nominal consumption expenditure ratios within each period depend on parameter θ. So, the individual decides on how much to spend on good 1 and good 2 during each period of his life on the basis of θ. Since

$$\frac{d}{d\theta}\left(\frac{\theta}{1-\theta}\right) = \frac{1}{(1-\theta)^2} > 0, \tag{3.21}$$

the higher the value of θ, the higher the expenditures on good 1 (the consumption-investment good) will be.

The fraction of income the individual consumes in the first period of his life is

$$\begin{aligned} \frac{c_{1yt} + p_t c_{2yt}}{w_t \bar{l}} &= \frac{\mu\theta w_t \bar{l} + \mu(1-\theta) w_t \bar{l}}{w_t \bar{l}} \\ &= \mu\theta + \mu(1-\theta) \\ &= \mu. \end{aligned} \tag{3.22}$$

Thus, the fraction of income saved is $(1-\mu)$. The fraction of income spent on good 1 during the first period is

$$\begin{aligned} \frac{c_{1t}}{w_t\bar{l}} &= \frac{\mu\theta w_t\bar{l}}{w_t\bar{l}} \\ &= \mu\theta, \end{aligned} \tag{3.23}$$

and the fraction of income spent on good 2 during the first period is

$$\begin{aligned} \frac{c_{2t}}{w_t\bar{l}} &= \frac{\mu(1-\theta)w_t\bar{l}}{w_t\bar{l}} \\ &= \mu(1-\theta). \end{aligned} \tag{3.24}$$

Therefore, μ determines the saving rate of this economy. In particular $(1-\mu)$ is the saving rate, which is constant and exogenously given. θ, on the other hand, determines the pattern of first period consumption. In particular, θ specifies the allocation of consumption expenditures during the first period, over the two goods.

3.2 Production

3.2.1 Profit Maximization Problem

Both the investment-consumption good and the consumption good are produced according to constant returns to scale Cobb-Douglas production technologies by using capital, K, and Labor, L. The output of the good 1, and that of good 2 at time t, X_{1t} and X_{2t}, are given by

$$X_{1t} = K_{1t}^{\alpha} L_{1t}^{1-\alpha}, \tag{3.25}$$

$$X_{2t} = K_{2t}^{\beta} L_{2t}^{1-\beta}. \tag{3.26}$$

In per capita terms

$$x_{1t} = k_{1t}^{\alpha} l_{1t}^{1-\alpha}, \tag{3.27}$$

$$x_{2t} = k_{2t}^{\beta} l_{2t}^{1-\beta}, \tag{3.28}$$

where

$$\begin{aligned} x_{it} &= \frac{X_{it}}{N_t}, \\ k_{it} &= \frac{K_{it}}{N_t}, \\ l_{it} &= \frac{L_{it}}{N_t}, \qquad \text{for} \qquad i = 1, 2. \end{aligned}$$

l_{it} is the proportion of labor force employed in sector i, at time t. Total labor supplied at time t is

$$L_t = N_t \bar{l},$$

where $\bar{l}$ is exogenously given and represents the level of labor supplied by an individual. When $\bar{l} = 1$, the per capita and the per worker transformations of the output functions are the same. Thus, $l_{it} \in [0, 1]$ is the proportion of the labor force employed in sector i at time t. Clearance of factor markets requires that

$$k_{1t} + k_{2t} = k_t, \tag{3.29}$$

and

$$l_{1t} + l_{2t} = \bar{l}. \tag{3.30}$$

The properties of the sectoral production technologies in association with the competitive nature of the economy imply that the demand for labor and capital in each sector is determined by first order conditions for profit maximization. If labor and capital are perfectly mobile across sectors and if both goods are produced, then

$$\begin{aligned} r_t &= \alpha k_{1t}^{\alpha-1} l_{1t}^{1-\alpha} \\ &= p_t \beta k_{2t}^{\beta-1} l_{2t}^{1-\beta}, \end{aligned} \tag{3.31}$$

$$\begin{aligned} w_t &= (1-\alpha) k_{1t}^{\alpha} l_{1t}^{-\alpha} \\ &= p_t (1-\beta) k_{2t}^{\beta} l_{2t}^{-\beta}. \end{aligned} \tag{3.32}$$

Where r_t is the rental rate on capital, w_t is the wage rate, and p_t is the price of the consumption good (good 2) in terms of the consumption-investment good (good 1), at time t. The consumption-investment good (good 1) is the numeraire. Side by side division of (3.31) and (3.32), results in

$$k_{1t} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1-\beta}{\beta}\right)\frac{l_{1t}}{l_{2t}} k_{2t}. \tag{3.33}$$

Using (3.31) and (3.33), we obtain

$$k_{1t} = p_t^{\frac{1}{\alpha-\beta}} \left(\frac{\beta}{\alpha}\right)^{\frac{\beta}{\alpha-\beta}} \left(\frac{1-\beta}{1-\alpha}\right)^{\frac{1-\beta}{\alpha-\beta}} l_{1t}. \tag{3.34}$$

Now, rearranging terms of (3.33) to obtain the expression of k_{2t} in terms of k_{1t}, and plugging in the expression for k_{1t} given by (3.34) yields

$$k_{2t} = p_t^{\frac{1}{\alpha-\beta}} \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha-\beta}} \left(\frac{1-\beta}{1-\alpha}\right)^{\frac{1-\alpha}{\alpha-\beta}} l_{2t}. \tag{3.35}$$

Let

$$\epsilon = \left(\frac{\beta}{\alpha}\right)^{\frac{\beta}{\alpha-\beta}} \left(\frac{1-\beta}{1-\alpha}\right)^{\frac{1-\beta}{\alpha-\beta}}, \tag{3.36}$$

and

$$\delta = \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha-\beta}} \left(\frac{1-\beta}{1-\alpha}\right)^{\frac{1-\alpha}{\alpha-\beta}}. \tag{3.37}$$

Hence,

$$k_{1t} = \epsilon l_{1t} p_t^{\frac{1}{\alpha-\beta}}, \tag{3.38}$$

and

$$k_{2t} = \delta l_{2t} p_t^{\frac{1}{\alpha-\beta}}. \tag{3.39}$$

Now, substituting (3.38) and (3.39) in the factor market clearing conditions; (3.29) and (3.30), yields

$$\begin{cases} \epsilon l_{1t} p_t^{\frac{1}{\alpha-\beta}} + \delta l_{2t} p_t^{\frac{1}{\alpha-\beta}} = k_t \\ l_{1t} + l_{2t} = \bar{l}. \end{cases} \tag{3.40}$$

Thus,

$$l_{1t} = \frac{\delta \bar{l}}{\delta - \epsilon} - \frac{1}{\delta - \epsilon} k_t p_t^{\frac{1}{\beta-\alpha}}, \tag{3.41}$$

$$l_{2t} = -\frac{\epsilon \bar{l}}{\delta - \epsilon} + \frac{1}{\delta - \epsilon} k_t p_t^{\frac{1}{\beta-\alpha}}. \tag{3.42}$$

Now, substituting (3.41) and (3.42) in (3.38) and (3.39) respectively, we get

$$k_{1t} = -\frac{\epsilon}{\delta - \epsilon} k_t + \frac{\delta \epsilon \bar{l}}{\delta - \epsilon} p_t^{\frac{1}{\alpha-\beta}}, \tag{3.43}$$

$$k_{2t} = \frac{\delta}{\delta - \epsilon} k_t - \frac{\delta \epsilon \bar{l}}{\delta - \epsilon} p_t^{\frac{1}{\alpha-\beta}}. \tag{3.44}$$

Now, using (3.31) and (3.38) the rental rate is given by

$$\begin{aligned} r_t &= \alpha \epsilon^{\alpha-1} p_t^{\frac{\alpha-1}{\alpha-\beta}} \\ &= \beta \delta^{\beta-1} p_t^{\frac{\alpha-1}{\alpha-\beta}}, \end{aligned} \tag{3.45}$$

and using (3.32) and (3.39) the wage rate is given by

$$\begin{aligned} w_t &= (1-\alpha) \epsilon^{\alpha} p_t^{\frac{\alpha}{\alpha-\beta}} \\ &= (1-\beta) \delta^{\beta} p_t^{\frac{\alpha}{\alpha-\beta}}. \end{aligned} \tag{3.46}$$

The expressions for output of good 1 is therefore determined by substituting (3.38) into (3.27) giving

$$x_{1t} = l_{1t}\epsilon^{\alpha} p_t^{\frac{\alpha}{\alpha-\beta}}, \tag{3.47}$$

and that of output of good 2 is determined by substituting (3.39) into (3.28) resulting in

$$x_{2t} = l_{2t}\delta^{\beta} p_t^{\frac{\beta}{\alpha-\beta}}. \tag{3.48}$$

Substituting (3.41) into (3.47), we get an expression for the output of good 1 in terms of the per capita capital and price ratio as follows

$$x_{1t} = \left(\frac{\delta \bar{l}}{\delta-\epsilon} - \frac{1}{\delta-\epsilon} k_t p_t^{\frac{1}{\beta-\alpha}}\right)\epsilon^{\alpha} p_t^{\frac{\alpha}{\alpha-\beta}}. \tag{3.49}$$

Similarly, substituting (3.42) into (3.48), we get that expression of the output of good 2 in terms of the per capita capital and the price ratio as follows,

$$x_{2t} = \left(-\frac{\epsilon \bar{l}}{\delta-\epsilon} + \frac{1}{\delta-\epsilon} k_t p_t^{\frac{1}{\beta-\alpha}}\right)\delta^{\beta} p_t^{\frac{\beta}{\alpha-\beta}}. \tag{3.50}$$

3.2.2 Parameters of the Production Technology

The sectoral constant returns to scale production technology is solely determined by the parameter α for sector 1 and β for sector 2. Since

$$\alpha = \frac{\partial x_{1t}(k_{1t}, l_{1t})}{\partial k_{1t}} \frac{k_{1t}}{x_{1t}}, \tag{3.51}$$

and

$$\beta = \frac{\partial x_{2t}(k_{2t}, l_{2t})}{\partial k_{2t}} \frac{k_{2t}}{x_{2t}}, \tag{3.52}$$

(3.51) and (3.52) respectively give the *elasticities* of output of good 1 and good 2 with respect to capital used. So, α (β) shows the change in output of good 1 (good 2) to result from a marginal change in capital used. Similarly, $(1-\alpha)$

and $(1-\beta)$ gives respectively, the *elasticities* of output of good 1 and good 2 with respect to labor used. Alternatively, these parameters can be viewed as the shares of respective factors of production in total cost.

3.3 The Autarky Economy

A perfect-foresight equilibrium is a sequence $\{k_t, p_t\}_{t=0}^{\infty}$ that clears the goods' markets at every period t while satisfying the dynamics of the capital stock at time $t+1$. The individual saves only during the first period of life. During the second period of life, the individual gets old and retires to consume all his wealth. The fraction of income saved during the first period of life is $(1-\mu)$. Thus the evolution of the per capita capital is governed by

$$k_{t+1} = \frac{(1-\mu)w_t\bar{l}}{(1+n)}. \tag{3.53}$$

The clearance of the goods' market in period t requires that per capita supply of each good be equal to its respective per capita demand. Hence, for good 1

$$x_{1t} + k_t = c_{1yt} + \frac{1}{(1+n)}c_{1ot} + (1+n)k_{t+1}, \tag{3.54}$$

and for good 2

$$x_{2t} = c_{2yt} + \frac{1}{(1+n)}c_{2ot}. \tag{3.55}$$

Applying Walras' Law allows us to focus on only one of the goods markets. So, we consider the market clearance condition for the consumption good (good 2).

Substituting (3.16) and (3.18) at t, in (3.55) yields

$$x_{2t} = \mu(1-\theta)\frac{w_t\bar{l}}{p_t} + \frac{(1-\mu)(1-\theta)}{1+n}(1+r_t)\frac{w_{t-1}\bar{l}}{p_t}. \tag{3.56}$$

Substituting (3.45) and (3.46) at time $t-1$ in (3.56) yields

$$\begin{aligned} x_{2t} &= \mu(1-\theta)(1-\beta)\bar{l}\delta^{\beta}p_t^{\frac{\beta}{\alpha-\beta}} \\ &+ \frac{(1-\mu)(1-\theta)(1-\beta)\bar{l}}{1+n}(1+\beta\delta^{\beta-1}p_t^{\frac{\alpha-1}{\alpha-\beta}})\frac{1}{p_t}\delta^{\beta}p_{t-1}^{\frac{\alpha}{\alpha-\beta}}. \end{aligned} \tag{3.57}$$

Substituting (3.48) in (3.57) and rearranging terms yield

$$\begin{aligned} l_{2t} &= \mu(1-\theta)(1-\beta)\bar{l} + \frac{1}{1+n}(1-\mu)(1-\theta)(1-\beta)\bar{l}p_{t-1}^{\frac{\alpha}{\alpha-\beta}}p_t^{\frac{-\alpha}{\alpha-\beta}} \\ &+ \frac{1}{1+n}(1-\mu)(1-\theta)(1-\beta)\beta\bar{l}\delta^{\beta-1}p_{t-1}^{\frac{\alpha}{\alpha-\beta}}p_t^{\frac{-1}{\alpha-\beta}}. \end{aligned} \tag{3.58}$$

Rearranging terms of (3.58) yields

$$\begin{aligned} l_{2t} &= \mu(1-\theta)(1-\beta)\bar{l} \\ &+ \frac{1}{1+n}(1-\mu)(1-\theta)(1-\beta)\bar{l}\left(\frac{p_{t-1}}{p_t}\right)^{\frac{\alpha}{\alpha-\beta}} \\ &+ \frac{1}{1+n}(1-\mu)(1-\theta)(1-\beta)\beta\bar{l}\delta^{\beta-1}p_{t-1}^{\frac{\alpha}{\alpha-\beta}}p_t^{\frac{-1}{\alpha-\beta}} \end{aligned} \tag{3.59}$$

Remembering (3.42)

$$l_{2t} = -\frac{\epsilon\bar{l}}{\delta-\epsilon} + \frac{1}{\delta-\epsilon}k_t p_t^{\frac{1}{\beta-\alpha}} \tag{3.60}$$

and substituting it in (3.59) leads to

$$\begin{aligned} k_t &= \{\mu(1-\theta)(1-\beta)(\delta-\epsilon)+\epsilon)\}\,\bar{l}p_t^{\frac{1}{\alpha-\beta}} \qquad (3.61) \\ &+ \frac{1}{1+n}(1-\mu)(1-\theta)(1-\beta)(\delta-\epsilon)\bar{l}p_{t-1}^{\frac{\alpha}{\alpha-\beta}}p_t^{\frac{1-\alpha}{\alpha-\beta}} \\ &+ \frac{1}{1+n}(1-\mu)(1-\theta)(1-\beta)(\delta-\epsilon)\beta\bar{l}\delta^{\beta-1}p_{t-1}^{\frac{\alpha}{\alpha-\beta}}. \end{aligned}$$

Let

$$\phi_1 = \{\mu(1-\theta)(1-\beta)(\delta-\epsilon)+\epsilon)\}\,\bar{l}, \tag{3.62}$$

$$\phi_2 = \frac{1}{1+n}(1-\mu)(1-\theta)(1-\beta)(\delta-\epsilon)\bar{l} \tag{3.63}$$

$$\phi_3 = \frac{1}{1+n}(1-\mu)(1-\theta)(1-\beta)(\delta-\epsilon)\beta\bar{l}\delta^{\beta-1}. \tag{3.64}$$

Hence,

$$k_t = \phi_1 p_t^{\frac{1}{\alpha-\beta}} + \phi_2 p_{t-1}^{\frac{\alpha}{\alpha-\beta}} p_t^{\frac{1-\alpha}{\alpha-\beta}} + \phi_3 p_{t-1}^{\frac{\alpha}{\alpha-\beta}}. \tag{3.65}$$

Now substituting the wage rate given by (3.46) into per capita capital dynamics given by (3.53) yields

$$k_{t+1} = \frac{1}{1+n}(1-\mu)(1-\beta)\bar{l}\delta^{\beta} p_t^{\frac{\alpha}{\alpha-\beta}}. \tag{3.66}$$

Let

$$\phi_4 = \frac{1}{1+n}(1-\mu)(1-\beta)\bar{l}\delta^{\beta}. \tag{3.67}$$

Thus,

$$k_{t+1} = \phi_4 p_t^{\frac{\alpha}{\alpha-\beta}}. \tag{3.68}$$

Writing (3.65) at time $t+1$ gives

$$k_{t+1} = \phi_1 p_{t+1}^{\frac{1}{\alpha-\beta}} + \phi_2 p_t^{\frac{\alpha}{\alpha-\beta}} p_{t+1}^{\frac{1-\alpha}{\alpha-\beta}} + \phi_3 p_t^{\frac{\alpha}{\alpha-\beta}}. \tag{3.69}$$

Now substituting (3.68) into (3.69) gives a nonlinear difference equation in terms of prices only that characterizes the dynamics of this economy:

$$(\phi_4 - \phi_3) p_t^{\frac{\alpha}{\alpha-\beta}} = \phi_1 p_{t+1}^{\frac{1}{\alpha-\beta}} + \phi_2 p_t^{\frac{\alpha}{\alpha-\beta}} p_{t+1}^{\frac{1-\alpha}{\alpha-\beta}}. \tag{3.70}$$

The dynamics of this model can also be characterized by a single nonlinear difference equation in terms of per capita capital only. The per capita capital dynamics given by (3.68) can be rewritten for p_t in terms of k_{t+1}, thus

$$p_t = \left(\frac{1}{\phi_4}\right)^{\frac{\alpha-\beta}{\alpha}} k_{t+1}^{\frac{\alpha-\beta}{\alpha}}. \tag{3.71}$$

Hence, using (3.71) and substituting in (3.65), one can obtain

$$(\phi_4 - \phi_3)k_t = \phi_1 \phi_4^{\frac{\alpha-1}{\alpha}} k_{t+1}^{\frac{1}{\alpha}} + \phi_2 \phi_4^{\frac{\alpha-1}{\alpha}} k_t k_{t+1}^{\frac{1-\alpha}{\alpha}}. \tag{3.72}$$

3.3.1 The Dynamic Equilibrium for the Autarky Economy

The dynamics of this economy can either be characterized by (3.70) or by (3.72). Considering only the price ratio dynamics governing this system in (3.70), we can start by solving for the steady state price ratio and determine the steady state magnitudes of the rest of variables. Now, (3.70) can be rewritten in such a way to facilitate the determination of the equilibrium price ratio. Since

$$\begin{aligned}
\phi_4 - \phi_3 &= \phi_1 \left(\frac{p_{t+1}^{\frac{1}{\alpha-\beta}}}{p_t^{\frac{\alpha}{\alpha-\beta}}} \right) + \phi_2 p_{t+1}^{\frac{1-\alpha}{\alpha-\beta}} \\
&= \phi_1 \left(\frac{p_{t+1}^{\frac{1}{\alpha-\beta}}}{p_t^{\frac{\alpha}{\alpha-\beta}}} \right) \left(\frac{p_{t+1}^{\frac{-\alpha}{\alpha-\beta}}}{p_{t+1}^{\frac{-\alpha}{\alpha-\beta}}} \right) + \phi_2 p_{t+1}^{\frac{1-\alpha}{\alpha-\beta}} \\
&= \left(\phi_1 \left(\frac{p_{t+1}}{p_t} \right)^{\frac{\alpha}{\alpha-\beta}} + \phi_2 \right) p_{t+1}^{\frac{1-\alpha}{\alpha-\beta}},
\end{aligned}$$

it follows that

$$p_{t+1} = \left(\frac{\phi_4 - \phi_3}{\phi_1 \left(\frac{p_{t+1}}{p_t} \right)^{\frac{\alpha}{\alpha-\beta}} + \phi_2} \right)^{\frac{\alpha-\beta}{1-\alpha}}. \tag{3.73}$$

Now, an equilibrium price ratio p_s (steady state value) is such that $p_{t+1} = p_t = p_s$ and satisfies (3.73). Then,

$$p_s = \left(\frac{\phi_4 - \phi_3}{\phi_1 + \phi_2} \right)^{\frac{\alpha-\beta}{1-\alpha}}. \tag{3.74}$$

Letting

$$\Phi = \frac{\phi_4 - \phi_3}{\phi_1 + \phi_2}, \tag{3.75}$$

one obtains

$$p_s = \Phi^{\frac{\alpha-\beta}{1-\alpha}}. \tag{3.76}$$

Similarly, considering (3.72) we can easily proceed to solve for the steady state magnitude of per capita capital k_s. Rearranging terms of (3.72), we have,

$$\begin{aligned}
\phi_4 - \phi_3 &= \phi_1 \phi_4^{\frac{\alpha-1}{\alpha}} \left(\frac{k_{t+1}^{\frac{1}{\alpha}}}{k_t} \right) + \phi_2 \phi_4^{\frac{\alpha-1}{\alpha}} k_{t+1}^{\frac{1-\alpha}{\alpha}} \\
&= \phi_1 \phi_4^{\frac{\alpha-1}{\alpha}} \left(\frac{k_{t+1}^{\frac{1}{\alpha}}}{k_t} \right) \left(\frac{k_{t+1}}{k_{t+1}} \right) + \phi_2 \phi_4^{\frac{\alpha-1}{\alpha}} k_{t+1}^{\frac{1-\alpha}{\alpha}} \\
&= \phi_4^{\frac{\alpha-1}{\alpha}} \left(\phi_1 \left(\frac{k_{t+1}}{k_t} \right) + \phi_2 \right) k_{t+1}^{\frac{1-\alpha}{\alpha}}.
\end{aligned}$$

Hence,

$$k_{t+1} = \phi_4 \left(\frac{\phi_4 - \phi_3}{\phi_1 \left(\frac{k_{t+1}}{k_t} \right) + \phi_2} \right)^{\frac{\alpha}{1-\alpha}}. \tag{3.77}$$

Now, an equilibrium k_s (steady state magnitude) is such that $k_{t+1} = k_t = k_s$ and satisfies (3.77). Then,

$$\begin{aligned}
k_s &= \phi_4 \left(\frac{\phi_4 - \phi_3}{\phi_1 + \phi_2} \right)^{\frac{\alpha}{1-\alpha}} \\
&= \phi_4 \Phi^{\frac{\alpha}{1-\alpha}}.
\end{aligned} \tag{3.78}$$

So, it is clear that the dynamics of this economy where production and utility are of the Cobb-Douglas type, are characterized by a single nonlinear difference equation in terms of either price ratio or per capita capital. This follows from the fact that savings are not affected by the rental rate (Galor (1992*a*)) making it possible for either of the difference equations characterizing the dynamics to be easily solved for the steady state magnitudes of relevant variables.

Proposition 1 *The equilibrium price ratio, p_s, for this perfect foresight overlapping-generations general equilibrium model with constant returns to scale* *production exists and is unique for all values of $-1 < n$, $\bar{l} > 0$ and for any given values of α, β, μ, θ that lie strictly between 0 and 1 such that $\alpha \neq \beta$ and initial per capita capital magnitude allowing for the attainment of the transition path.*

Proof:

The long-run closed form solution given in (3.74) shows that p_s is unique. However, existence must be assured by showing that p_s is positive for any α, β, μ, θ, n and $\bar{l}$.

For p_s to be positive, Φ must be positive. $\Phi = \frac{\phi_4 - \phi_3}{\phi_1 + \phi_2} > 0$ if and only if $\phi_4 - \phi_3 > 0$ and $\phi_1 + \phi_2 > 0$ or $\phi_4 - \phi_3 < 0$ and $\phi_1 + \phi_2 < 0$.

Now, $\phi_4 - \phi_3 > 0 \Leftrightarrow$

$$\begin{aligned}
\frac{1}{1+n}(1-\mu)(1-\beta)\bar{l}\delta^{\beta} &> \frac{1}{1+n}(1-\mu)(1-\theta)(1-\beta)\beta\bar{l}(\delta-\epsilon)\delta^{\beta-1} \\
\delta^{\beta} &> (1-\theta)\beta(\delta-\epsilon)\delta^{\beta-1} \\
1 &> (1-\theta)\beta(1-\frac{\epsilon}{\delta}) \\
1 &> (1-\theta)\beta\left(1-\left(\frac{1-\beta}{1-\alpha}\right)\left(\frac{\alpha}{\beta}\right)\right) \\
1 &> \frac{(1-\theta)(\beta-\alpha)}{(1-\alpha)} \\
(1-\alpha)+\alpha(1-\theta) &> (1-\theta)\beta \\
\frac{1-\alpha\theta}{1-\theta} &> \beta.
\end{aligned}$$

Since $\frac{1-\alpha\theta}{1-\theta} > 1$, and $0 < \beta < 1$,

$$\frac{1-\alpha\theta}{1-\theta} > \beta$$

holds for any given α, β, and θ. Thus $\phi_4 - \phi_3 > 0$ for any given α, β, μ, θ, n and $\bar{l}$. Similarly $\phi_1 + \phi_2 > 0$ can be shown as follows:

$\phi_1 + \phi_2 > 0 \Leftrightarrow$

$$\begin{aligned}
\mu(1-\theta)(1-\beta)\bar{l}(\delta-\epsilon) + \epsilon\bar{l} + \frac{1}{1+n}(1-\mu)(1-\theta)(1-\beta)\bar{l}(\delta-\epsilon) &> 0 \\
(\delta-\epsilon)\left(\mu(1-\theta)(1-\beta) + \frac{1}{1+n}(1-\mu)(1-\theta)(1-\beta)\right) + \epsilon &> 0 \\
\left(\frac{\delta}{\epsilon} - 1\right)(1-\theta)(1-\beta)\left(\mu + \frac{1-\mu}{1+n}\right) &> -1 \\
\left(\frac{\beta-\alpha}{\alpha}\right)(1-\theta)\left(\frac{1+\mu n}{1+n}\right) &> -1 \\
-\frac{1+n}{(1+\mu n)(1-\theta)} &< \frac{\beta-\alpha}{\alpha} \\
1 - \frac{1+n}{(1+\mu n)(1-\theta)} &< \frac{\beta}{\alpha}.
\end{aligned}$$

Since

$$\frac{1+n}{(1+\mu n)(1-\theta)} > 1,$$

$$1 - \frac{1+n}{(1+\mu n)(1-\theta)} < 0.$$

Given that $\frac{\beta}{\alpha} > 0$,

$$\frac{\beta}{\alpha} > 1 - \frac{1+n}{(1+\mu n)(1-\theta)}$$

holds for any given values of α, β, μ, θ, $\bar{l}$, and n.

Hence $\phi_1 + \phi_2 > 0$ for any given α, β, μ, θ, n and $\bar{l}$.

Therefore, $\Phi > 0$ and hence $p_s > 0$ for any given α, β, μ, θ, n and $\bar{l}$, where $0 < \alpha < 1$, $0 < \beta < 1$, $0 < \mu < 1$, $0 < \theta < 1$, $0 < \bar{l}$, and $-1 < n$. ∎

It must be noted that for (3.74) to hold, it suffices to set $\alpha \neq \beta$, without specifying whether $\alpha > \beta$ or $\alpha < \beta$. In other words, it is not necessary to restrict α to be greater than β so as to let the production of good 1 be relatively capital intensive. This is an important finding complementing the restrictive set of conditions put forth by Galor (1992a) who states that the perfect-foresight equilibrium of a two sector overlapping-generations model is globally unique if all of the following hold:

1. the investment good is capital intensive,
2. first and second period consumption are gross substitutes (i.e., the saving is an increasing function of the real rate of return to capital), and
3. second period consumption is a normal good.

Our solutions to the model we develop by allowing the investment good to serve for consumption purposes as well, demonstrate therefore that neither the first nor the second item is required to guarantee the existence of a unique global steady state solution.

3.3.2 Long-run Closed Form Solutions for the Autarky Economy

Now, given the price ratio p_s we can easily proceed to find out the closed form solutions for the steady state magnitudes of other model variables. Alternatively, one can use (3.68) to obtain the steady state magnitude k_s of per capita capital and proceed to solve for other variables:

$$k_{t+1} - k_t = \phi_4 p_t^{\frac{\alpha}{\alpha-\beta}} - k_t. \tag{3.79}$$

The steady state magnitude k_s is such that $k_{t+1} - k_t = 0$. Hence,

$$k_s = \phi_4 p_s^{\frac{\alpha}{\alpha-\beta}}. \tag{3.80}$$

Thus, the steady state per capita capital is

$$\begin{aligned} k_s &= \phi_4 \left(\frac{\phi_4 - \phi_3}{\phi_1 + \phi_2} \right)^{\frac{\alpha}{1-\alpha}} \\ &= \phi_4 \Phi^{\frac{\alpha}{1-\alpha}} \end{aligned} \tag{3.81}$$

Using (3.46), one can obtain the following expression for w_s,

$$w_s = (1-\alpha)\epsilon^{\alpha} p_s^{\frac{\alpha-1}{\alpha-\beta}}. \tag{3.82}$$

Substituting (3.76) into (3.82), we easily obtain the expression for the steady state wage rate w_s,

$$w_s = (1-\alpha)\epsilon^{\alpha} \Phi^{\frac{\alpha}{\alpha-\beta}}. \tag{3.83}$$

Using (3.45), we can obtain the following expression for r_s

$$r_s = \alpha \epsilon^{\alpha-1} p_s^{\frac{\alpha-1}{\alpha-\beta}}. \tag{3.84}$$

Substituting (3.76) into (3.84), we obtain the steady state rental rate as

$$r_s = \alpha \epsilon^{\alpha-1} \frac{1}{\Phi}. \tag{3.85}$$

The steady state expression for the output of good 1 is determined by substituting (3.76) and (3.81) into (3.49) giving

$$x_{1s} = \frac{\delta \bar{l}}{\delta - \epsilon} \epsilon^{\alpha} \Phi^{\frac{\beta}{1-\alpha}} + \frac{1}{\delta - \epsilon} \delta^{\beta} \phi_4 \Phi^{\frac{2\alpha-1}{1-\alpha}}. \tag{3.86}$$

The steady state expression for the output of good 2 is determined by substituting (3.76) and (3.81) into (3.50) giving

$$x_{2s} = -\frac{\epsilon \bar{l}}{\delta - \epsilon} \delta^{\beta} \Phi^{\frac{\beta}{1-\alpha}} + \frac{1}{\delta - \epsilon} \delta^{\beta} \phi_4 \Phi^{\frac{\alpha+\beta-1}{1-\alpha}}. \tag{3.87}$$

The steady state real consumption by young can be obtained once the steady state wage rate is determined. Using (3.15), we have

$$c_{1ys} = \mu \theta w_s \bar{l}. \tag{3.88}$$

Hence, substituting (3.83) into (3.88) yields

$$c_{1ys} = \mu \theta (1 - \alpha) \bar{l} \epsilon^{\alpha} \Phi^{\frac{\alpha}{1-\alpha}}. \tag{3.89}$$

(3.16) evaluated at the steady state leads to

$$c_{2ys} = \mu (1 - \theta) \frac{w_s \bar{l}}{p_s}. \tag{3.90}$$

Substituting (3.76) and (3.83) into (3.90) yields

$$c_{2ys} = \mu (1 - \theta)(1 - \alpha) \bar{l} \epsilon^{\alpha} \Phi^{\frac{\beta}{1-\alpha}}. \tag{3.91}$$

The steady state real consumption by old can be determined once the steady state price ratio, the steady state wage rate and the steady state rental rate are determined. Using (3.17), we have

$$c_{1os} = (1 - \mu) \theta (1 + r_s) w_s \bar{l}. \tag{3.92}$$

Substituting (3.83) into (3.92), we get

$$c_{1os} = (1 - \mu) \theta (1 - \alpha) \bar{l} \epsilon^{\alpha} (1 + \alpha \epsilon^{\alpha - 1} \frac{1}{\Phi}) \Phi^{\frac{\alpha}{1-\alpha}}. \tag{3.93}$$

Using (3.18), we have

$$c_{2os} = (1-\mu)(1-\theta)(1+r_s)\frac{w_s \bar{l}}{p_s}. \tag{3.94}$$

Substituting (3.76) and (3.83) into (3.94), we get

$$c_{2os} = (1-\mu)(1-\theta)(1-\alpha)\bar{l}\epsilon^{\alpha}(1+\alpha\epsilon^{\alpha-1}\frac{1}{\Phi})\Phi^{\frac{\beta}{1-\alpha}}. \tag{3.95}$$

CHAPTER IV

THE BASIC MODEL AT WORK

In order to visualize the dynamics of our model economy, a numerical exercise is worth undertaking. Such an exercise is likely to produce results that help us understand the significance and implications for growth of different initial conditions and parameter values. We begin by assigning numerical values to parameters and consider a set of initial magnitudes for per capita capital and price ratios. The numerical solutions enable us to observe paths that different variables would follow over time until the steady state magnitudes analytically derived in the previous chapter are reached (asymptotically with a convergence margin smaller than 10^{-6}). We also discuss the possibility of non-convergence observed under certain initial values and try to generalize our results to establish conditions for such non-convergence. More precisely, we have noticed that for some initial values the economy stays stack in its initial variable magnitudes, but once the transition path is reached, the long-run equilibrium magnitudes are reachable.

Production parameters are chosen such that the production of commodity 1 is relatively capital-intensive, and that of commodity 2 is relatively labor-intensive as suggested by Galor (1992*b*). Hence, the condition

$$\frac{K_{1t}}{L_{1t}} > \frac{K_{2t}}{L_{2t}},$$

must be satisfied at any time t, where K_{it} and L_{it} for $i = 1, 2$, are respectively the amounts of capital and the labor employed in the production of good i. Because markets are competitive, both capital and labor earn their marginal products. Since capital and labor are perfectly mobile across sectors, the rental rate and the wage rate are equalized across sectors. Hence, marginal product of each factor in sector 1 needs to be equated to its marginal product in sector 2.

So,

$$\begin{aligned} \frac{\partial X_{1t}(K_{1t}, L_{1t})}{\partial K_{1t}} &= \frac{\partial X_{2t}(K_{2t}, L_{2t})}{\partial K_{2t}}, \\ \Rightarrow \alpha K_{1t}^{\alpha-1} L_{1t}^{1-\alpha} &= p_t \beta K_{2t}^{\beta-1} L_{2t}^{1-\beta}. \end{aligned} \tag{4.1}$$

Similarly,

$$\begin{aligned} \frac{\partial X_{1t}(K_{1t}, L_{1t})}{\partial L_{1t}} &= \frac{\partial X_{2t}(K_{2t}, L_{2t})}{\partial L_{2t}}, \\ \Rightarrow (1-\alpha) K_{1t}^{\alpha} L_{1t}^{-\alpha} &= p_t (1-\beta) K_{2t}^{\beta} L_{2t}^{-\beta}. \end{aligned} \tag{4.2}$$

Side-by-side division of (4.2) and (4.1) yields

$$\frac{K_{1t}}{L_{1t}} = \left(\frac{\alpha}{\beta}\right) \left(\frac{1-\beta}{1-\alpha}\right) \frac{K_{2t}}{L_{2t}}. \tag{4.3}$$

In order to have

$$\frac{K_{1t}}{L_{1t}} > \frac{K_{2t}}{L_{2t}}$$

Table 4.1: Autarky Model Parameter Values

α	β	μ	θ	$\bar{l}$
0.50	0.30	0.80	0.40	1

holding at any time t, we need to have

$$\begin{aligned} \left(\frac{\alpha}{\beta}\right)\left(\frac{1-\beta}{1-\alpha}\right) &> 1 \\ \Rightarrow \alpha &> \beta. \end{aligned} \tag{4.4}$$

Therefore, having (4.4) satisfied guarantees that sector 1 is relatively capital intensive while sector 2 is relatively labor intensive in line with Galor (1992*b*). The parameter values chosen for numerical solution of the model are displayed in Table 4.1.

The population growth rate is taken to be $n = 0.16$, the equivalent of a 0.05 annual population growth rate, compounded over a 30 years period. By these numbers, the share of capital in the production of good 1 is 0.50 and that in the production of good 2 is 0.30. In a study conducted by Chenery (1986), the estimates of capital share vary considerably across countries ranging from 26% for Hundaras to more than 60% for Singapore. He also derived an estimate for the unweighted average of capital share for industrial countries, which is found to be about 30%. The fraction of wage income that is saved is assumed to be 0.20 following Loayza, Lopez and Serven (1998) who found that the unweighted average of the private saving rate for industrial countries is about 20%. These values imply that the fraction of wage income spent on first period consumption of good 1 is 0.32, and the fraction of income spent on first period consumption of good 2 is 0.48.

These parameter values imply the following values for ϕ_1 to ϕ_4. $\phi_1 = 1.2192$; $\phi_2 = -0.0624$; $\phi_3 = -0.0254$; $\phi_4 = 0.1059$.

The resulting dynamics can be analyzed by considering (3.70). Figure 4.1 shows p_{t+1} as a function of p_t. This is derived from (3.70) as follows: First remembering (3.70), we have

$$(\phi_4 - \phi_3) p_t^{\frac{\alpha}{\alpha-\beta}} = \phi_1 p_{t+1}^{\frac{1}{\alpha-\beta}} + \phi_2 p_t^{\frac{\alpha}{\alpha-\beta}} p_{t+1}^{\frac{1-\alpha}{\alpha-\beta}}.$$

We can rewrite this as

$$\left(\phi_4 - \phi_3 - \phi_2 p_{t+1}^{\frac{1-\alpha}{\alpha-\beta}}\right) p_t^{\frac{\alpha}{\alpha-\beta}} = \phi_1 p_{t+1}^{\frac{1}{\alpha-\beta}},$$

or,

$$\begin{aligned} p_t &= \left(\frac{\phi_1 p_{t+1}^{\frac{1}{\alpha-\beta}}}{\phi_4 - \phi_3 - \phi_2 p_{t+1}^{\frac{1-\alpha}{\alpha-\beta}}}\right)^{\frac{\alpha-\beta}{\alpha}} \\ &= \left(\frac{\phi_4 - \phi_3 - \phi_2 p_{t+1}^{\frac{1-\alpha}{\alpha-\beta}}}{\phi_1 p_{t+1}^{\frac{1}{\alpha-\beta}}}\right)^{\frac{\beta-\alpha}{\alpha}} \\ &= \left(\frac{\phi_4 - \phi_3}{\phi_1} p_{t+1}^{\frac{1}{\beta-\alpha}} - \frac{\phi_2}{\phi_1} p_{t+1}^{\frac{\alpha}{\beta-\alpha}}\right)^{\frac{\beta-\alpha}{\alpha}}. \end{aligned} \tag{4.5}$$

The relationship between p_{t+1} and p_t captured through (4.5) is plotted in Figure 4.1 for given values of parameters.

At the point of intersection between p_{t+1} and the 45-degree line, p_{t+1} equals p_t. It is clear that there is a unique equilibrium level of p_t (aside from $p = 0$), which we denote by p_s. The steady state price ratio p_s is globally stable: wherever the price ratio p starts (other than at 0), it converges to p_s. Suppose, for example, that the initial price ratio is p_0 which is less than p_s (i.e., to the left of p_s). Because p_{t+1} is greater than p_t when p_t is less than p_s, p_1 is greater than p_0. In the next

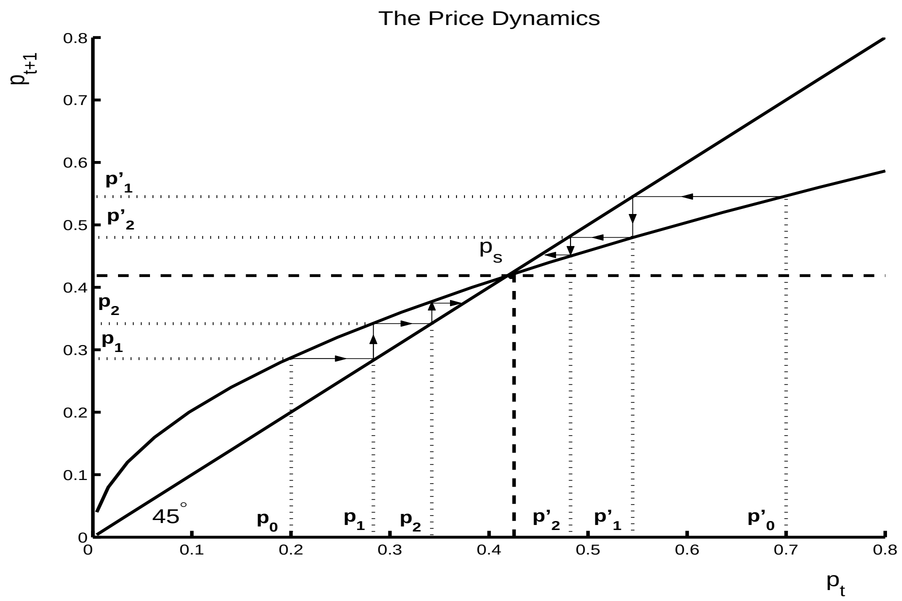

Figure 4.1: Dynamics of the Price Ratio

period, p_1 becomes the current period price. (4.5) indicates that the price must continue to increase. Since $p_2 \neq p_1$, p_1 can not be equal to p_s. Furthermore, since $p_2 > p_1$, p_1 must lie to the left of p_s, or $p_1 < p_s$. So, $p_0 < p_1 < p_s$ implying that the movement from p_0 to p_1 represents a part way step towards p_s. This process is repeated each period, and p converges smoothly to p_s. A similar analysis applies for initial price ratios greater than p_s such as p_0' (i.e., to the right of p_s). Hence, the initial price ratio affects only the transition path, but not the equilibrium magnitudes. Hence, the steady state price ratio exists and is unique as analytically shown in the previous chapter. Moreover, the equilibrium price sequence is unique and stationary.

Similarly, analyzing the model economy's dynamics through (3.72) results in the same conclusions in terms of per capita capital as it is displayed in Figure 4.2.

The relationship between k_{t+1} and k_t plotted in Figure 4.2 is derived through the following steps. Remembering (3.72)

$$(\phi_4 - \phi_3)k_t = \phi_1\phi_4^{\frac{\alpha-1}{\alpha}} k_{t+1}^{\frac{1}{\alpha}} + \phi_2\phi_4^{\frac{\alpha-1}{\alpha}} k_t k_{t+1}^{\frac{1-\alpha}{\alpha}}. \tag{4.6}$$

Rewriting the above as

$$\left(\phi_4 - \phi_3 - \phi_2\phi_4^{\frac{\alpha-1}{\alpha}} k_{t+1}^{\frac{1-\alpha}{\alpha}}\right) k_t = \phi_1\phi_4^{\frac{\alpha-1}{\alpha}} k_{t+1}^{\frac{1}{\alpha}},$$

one obtains

$$\begin{aligned} k_t &= \left(\frac{\phi_1\phi_4^{\frac{\alpha-1}{\alpha}} k_{t+1}^{\frac{1}{\alpha}}}{\phi_4 - \phi_3 - \phi_2\phi_4^{\frac{\alpha-1}{\alpha}} k_{t+1}^{\frac{1-\alpha}{\alpha}}}\right) \\ &= \left(\frac{\phi_4 - \phi_3 - \phi_2\phi_4^{\frac{\alpha-1}{\alpha}} k_{t+1}^{\frac{1-\alpha}{\alpha}}}{\phi_1\phi_4^{\frac{\alpha-1}{\alpha}} k_{t+1}^{\frac{1}{\alpha}}}\right)^{-1} \\ &= \left(\frac{\phi_4 - \phi_3}{\phi_1\phi_4^{\frac{\alpha-1}{\alpha}}} k_{t+1}^{-\frac{1}{\alpha}} - \frac{\phi_2}{\phi_1} k_{t+1}^{-1}\right)^{-1}, \end{aligned} \tag{4.7}$$

which is what is plotted in Figure 4.2.

k_s is globally stable: wherever k starts (other than 0), it converges to k_s. Suppose that k is initially at a magnitude such as k'_0, greater than k_s (i.e., to the right of k_s). Because k_{t+1} is less than k_t when k_t exceeds k_s, k'_1 is less than k'_0. By a similar line of reasoning to that employed in the context of the relationship between p_{t+1} and p_t, k'_1 should lie between k_s and k'_0. Thus, at every step k moves part way towards k_s. This process gets repeated in each period resulting in k converging smoothly to k_s. A similar analysis applies when the initial per capita capital magnitude is less than k_s such as k_0.

In order to analyze the global dynamics of our system, the phase diagram in Figure 4.3 is developed.

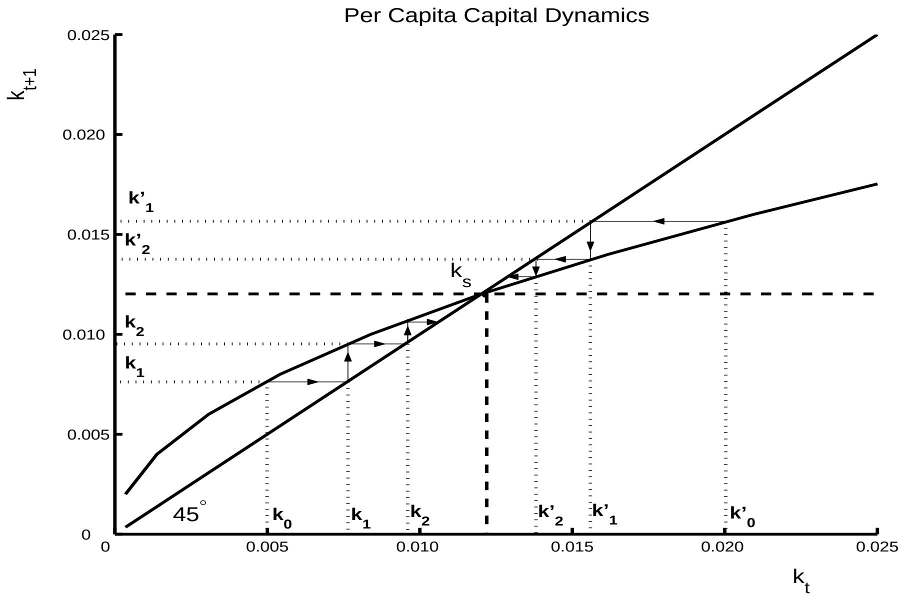

Figure 4.2: The Dynamics of Per Capita Capital

The phase diagram in Figure 4.3 locates (k_t, p_t) pairs at which per capita capital or the price ratio stop changing over time.

Per capita capital at period $t+1$ is equal to savings at period t which is given by (3.66). Thus subtracting k_t from both sides of (3.66) yields

$$k_{t+1} - k_t = \phi_4 p_t^{\frac{\alpha}{\alpha-\beta}} - k_t.$$

When $k_{t+1} - k_t = 0$, $k_t = \phi_4 p_t^{\frac{\alpha}{\alpha-\beta}}$. Thus, this equation captures the relationship between p_t and k_t when $k_{t+1} - k_t = 0$, and is plotted as the $\Delta k_t = 0$ curve in Figure 4.3. This curve shows possible steady state magnitudes of per capita capital for a given price ratio. Points lying to the right of (below) the curve show that k_t is greater than the steady state magnitude implied by the given price ratio. As Figure 4.2 shows when k_t is greater than k_s, $k_{t+1} < k_t$ or $\Delta k_t < 0$, implying that k would be declining over time. Similarly, for points lying to the left of (above) this curve, $k_{t+1} > k_t$ or $\Delta k_t > 0$.

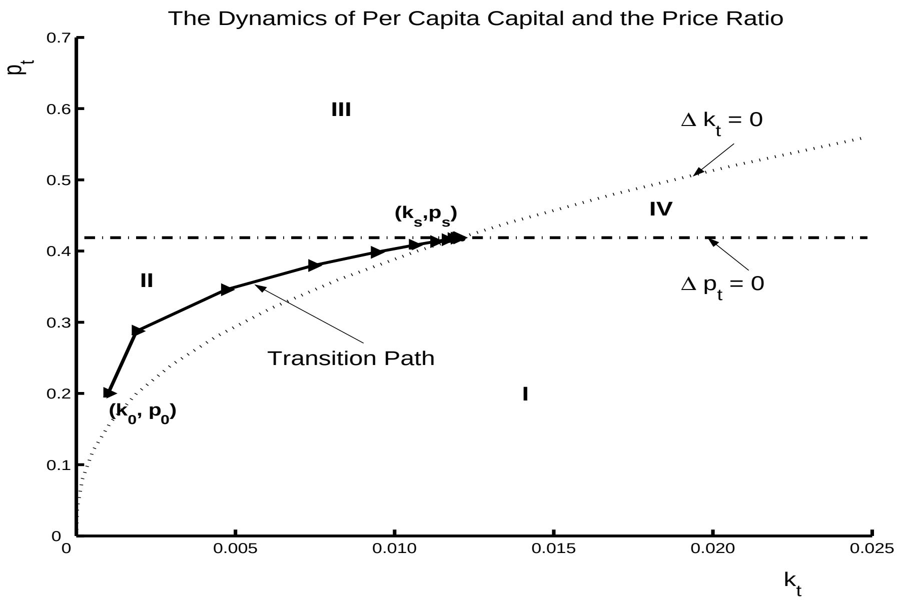

Figure 4.3: Phase Diagram for Per Capita Capital and the Price Ratio

The locus of (k_t, p_t) pairs at which the price ratio is at its steady state, is given by p_s from (3.76). This is the constant $\Delta p_t = 0$ plot in Figure 4.3. Points lying below this horizontal line correspond to price ratios smaller than p_s for which $p_{t+1} > p_t$ or $\Delta p_t > 0$ by Figure 4.1. Likewise, points above this line indicate that $p_{t+1} < p_t$ by the same figure.

Region I of the phase diagram contains points below both the $\Delta k_t = 0$ and the $\Delta p_t = 0$ loci. Hence, $\Delta k_t < 0$ and $\Delta p_t > 0$, implying that this is the region where k_t is falling and p_t is rising. Points in Region II are above the $\Delta k_t = 0$ locus but below the $\Delta p_t = 0$ locus. Hence, $\Delta k_t > 0$ and $\Delta p_t > 0$ or, both k_t and p_t are rising. Points in Region III are above both the $\Delta k_t = 0$ locus and the $\Delta p_t = 0$ locus. With $\Delta k_t > 0$ and $\Delta p_t < 0$, k_t is rising and p_t is falling. Finally, region IV includes points below $\Delta k_t = 0$ and above the $\Delta p_t = 0$ locus. Hence, $\Delta k_t < 0$ and $\Delta p_t < 0$. Thus, both k_t and p_t are falling. Therefore, whatever magnitudes per capita capital and the price ratio initially take, the same steady state (k_s, p_s) is reached.

The numerical values behind Figure 4.1 and Figure 4.2 (reported in Table 4.1) initially places the economy in region II of the phase diagram in Figure 4.3. These values are chosen to make sure that the initial price ratio and the initial per capita capital magnitude would put the economy on its transition path towards the steady state. Under these circumstances, per capita capital and the price ratio would steadily increase until the steady state is reached. Yet, the same would not necessarily be valid for all initial price ratios and initial per capita capital magnitudes. In general, the ability of the economy to converge towards the long-run equilibrium would depend on these initial values.

To investigate the effects of initial values on convergence, different initial per capita capital magnitudes (k) and different initial price ratios (p) were considered and the associated number of iterations required for the numerical steady state solution to be reached were computed. The results are reported below.

Table 4.2: Initial Price Ratios and Per Capita Capital

	k_0	p_0	i^*	
I_0	0.0200	0.3000	11	Region I
I_1	0.0050	0.1000	13	
I_2	0.0030	0.2500	11	
I_3	0.0010	0.2700	11	Region II
I_4	0.0035	0.3500	10	
I_5	0.0085	0.4187	1	$\Delta p_t = 0$ locus
I_6	0.0120	0.4500	8	Region III
I_7	0.0200	0.4187	8	Region IV
I_8	0.0200	0.4500	10	
I_9	0.0200	0.4900	1	$\Delta p_t = 0$ locus

* i is the number of moves from the initial values to the steady state.

For different initial price ratios and per capita capital magnitudes considered, the transition paths of the economy are plotted in Figure 4.4 through Figure 4.7.

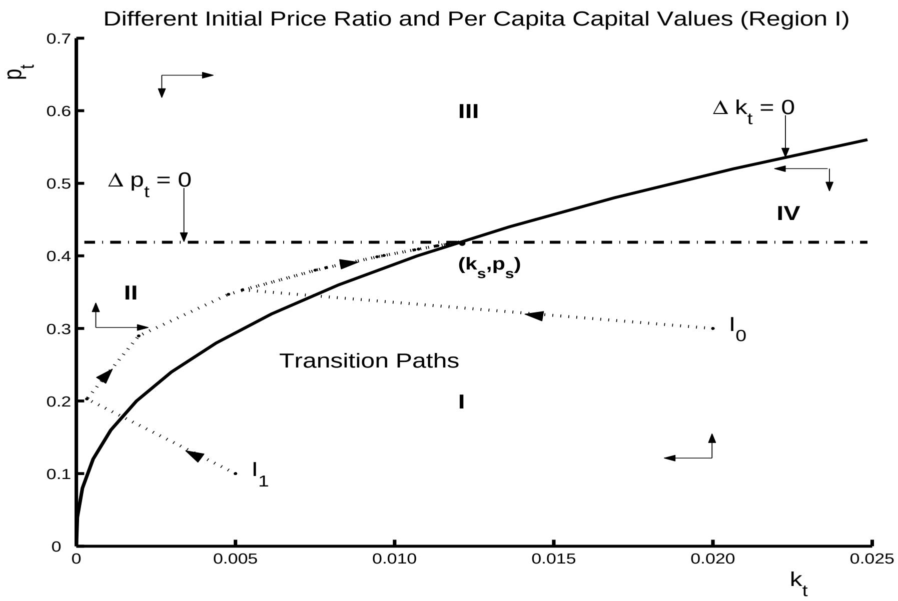

Figure 4.4: Phase Diagram for Per Capita Capital and the Price Ratio for Different Initial Values in Region I

It is clearly observed that these transition paths have a common part which leads the economy towards the steady state. For any initial (k_0, p_0) pair, the economy converges to the steady state, if the transition curve is reachable from those initial values while respecting the dynamics.

According to Figure 4.3, the economy can initially be in one of four different regions. If the economy starts from a point in region I, such as point I_0 or I_1 in Figure 4.4, then a one period move occurs in the price ratio causing per capita capital to cross the $\Delta k_t = 0$ locus, and the economy settles in region I after which the economy moves gradually to the steady state with the price ratio as well as per capita capital increasing. Hence, for any initial values in region I, the economy converges to its steady state solution.

If the economy starts from a point in region II, such as point I_2 or I_3 in Figure 4.5, then the economy moves gradually towards the steady state solution.

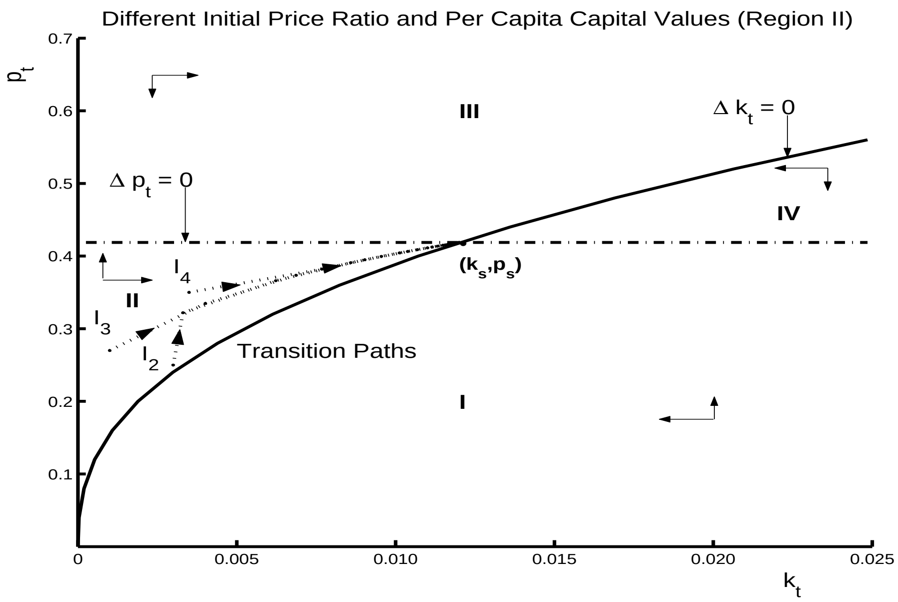

Figure 4.5: Phase Diagram for Per Capita Capital and the Price Ratio for Different Initial Values in Region II

However, for other points of region II that lie above the transition curve, the steady state solution is not always reachable and the economy stays stuck in its initial conditions. More precisely, starting with such points as point I_4 from which the economy can be put on the transition curve by an increase in both the price ratio and per capita capital magnitude, the steady state solution will be reachable. Otherwise, steady state solution will be impossible to reach. Hence, for all initial values in region II that lie below the transition curve, the economy converges to its steady state solution. However, for only some other points that lie above the transition curve, the economy converges to its steady state solution.

If the economy starts from a point in region III, then it may not always converge to its steady state solution, staying stuck in its initial conditions instead. In fact, in this region per capita capital is increasing while the price ratio is decreasing. So, an increase in per capita capital is needed in order to put the economy on the transition curve. However, this is not allowed because of the autarkic nature

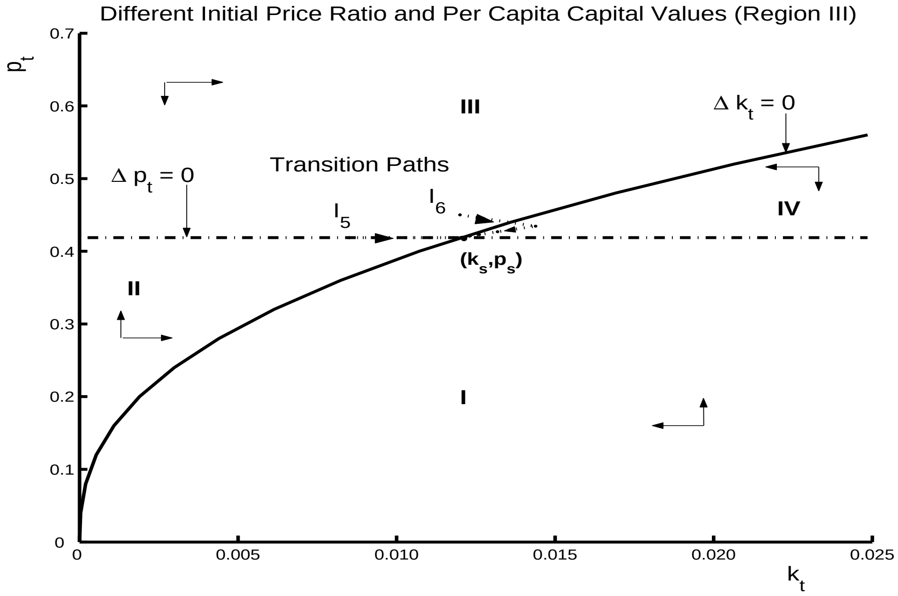

Figure 4.6: Phase Diagram for Per Capita Capital and the Price Ratio for Different Initial Values in Region III

of the economy which rules out the possibility of capital inflows from the outside world. In other words, since the only source of capital is the savings of the young, capital initially available to the economy may not be sufficient to put it on the transition curve. The only points of region III from which the economy can converge to its steady state are those that are close enough to (k_s, p_s), such as I_6 in Figure 4.6. So, initially per capita capital increases and price ratio decreases and puts the economy in region IV. Then, the economy is governed by the dynamics of region IV where per capita capital decreases and the price ratio deceases, until the steady state is reached. For some other points of region III that lie just a little bit above the steady state price ratio the economy converges, if the available capital is sufficient to allow for the existence of a general equilibrium solution.

If the economy starts from some point lying to the left of (k_s, p_s) and on the $\Delta p_t = 0$ locus, such as point I_5 in Figure 4.6, then it is not always the case that the economy converges. From points to the right of I_5, the economy converges in a one period move to the steady state. From points to the left of I_5, on the other hand, the economy can not converge since the available capital is not sufficient and there are no additional means of accumulating extra capital. Hence, the economy gets stuck in its initial conditions where a general equilibrium solution does not exist.

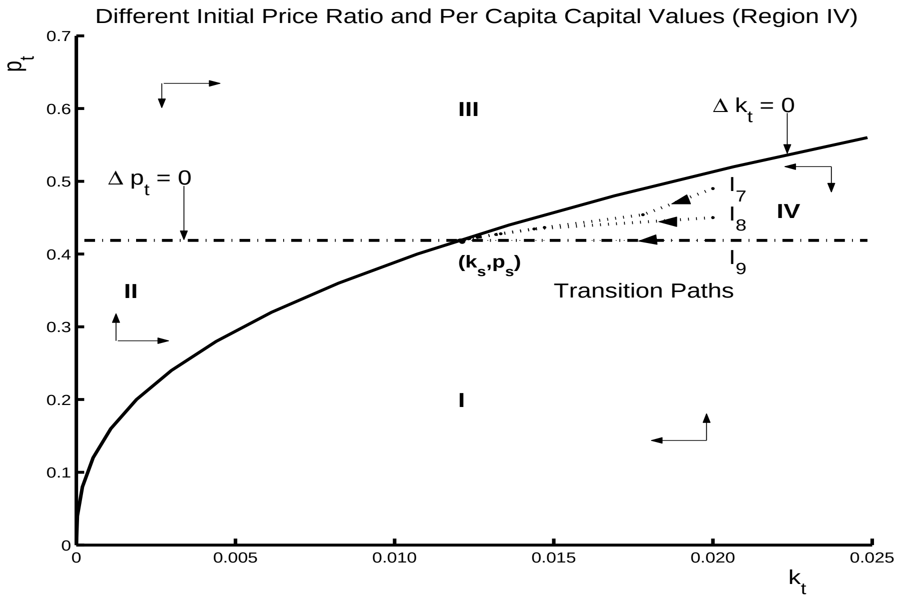

Figure 4.7: Phase Diagram for Per Capita Capital and the Price Ratio for Different Initial Values in Region IV

If the economy starts from a point in region IV, such as point I_8 or I_9 in Figure 4.7, then the economy moves gradually towards the steady state solution by a decrease in both the price ratio and per capita capital. In fact, for all points of region IV, the economy converges to the steady state solution. In fact, in this region more capital is available than enough to have producers maximizing profits, consumers maximizing utility and markets cleared.

If the economy starts from some point lying to the right of (k_s, p_s) and on the $\Delta p_t = 0$ locus, such as point I_7 in Figure 4.7, then it is always the case that the economy converges to the steady state, since there is already enough capital.

Returning to the numerical values we considered in Table4.1, the time paths of per capita capital and the price ratio are given in Figure 4.8. As it is clearly seen, per capita capital starts increasing until stabilizing at its steady state magnitude. A similar observation holds for the price ratio.

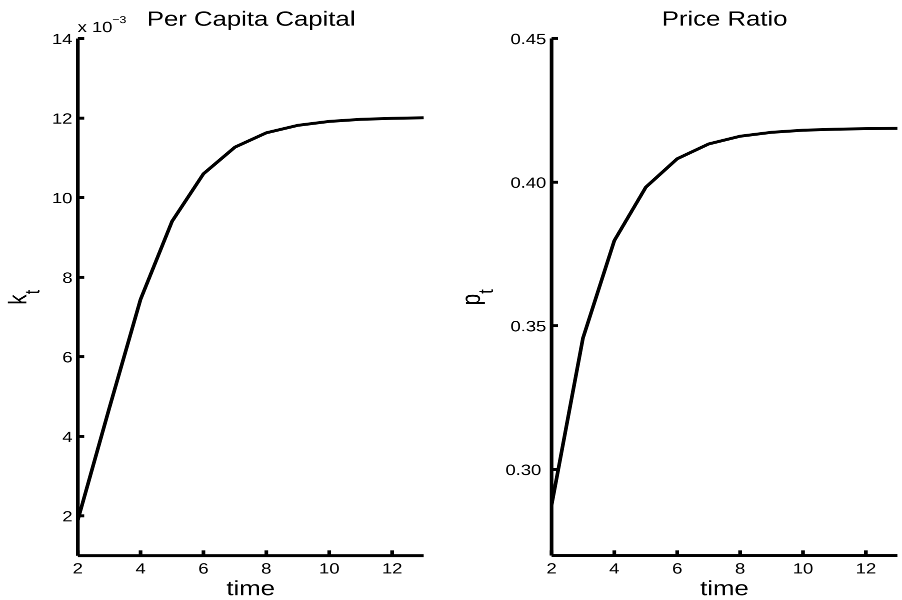

Figure 4.8: Time Paths for Per Capita Capital and the Price Ratio

Table 4.3: Steady State of k and p

k_0	p_0	k_s	p_s
0.0010	0.2000	0.0120	0.4187

The long-run equilibrium magnitude of per capita capital and the price ratio for this numerical example are given in Table 4.3.

The time paths of the rental and wage rates are given in Figure 4.9. Since per capita capital is increasing, it is expected that the rental rate will be decreasing and the wage rate will be increasing. This is exactly what is observed from Figure 4.9.

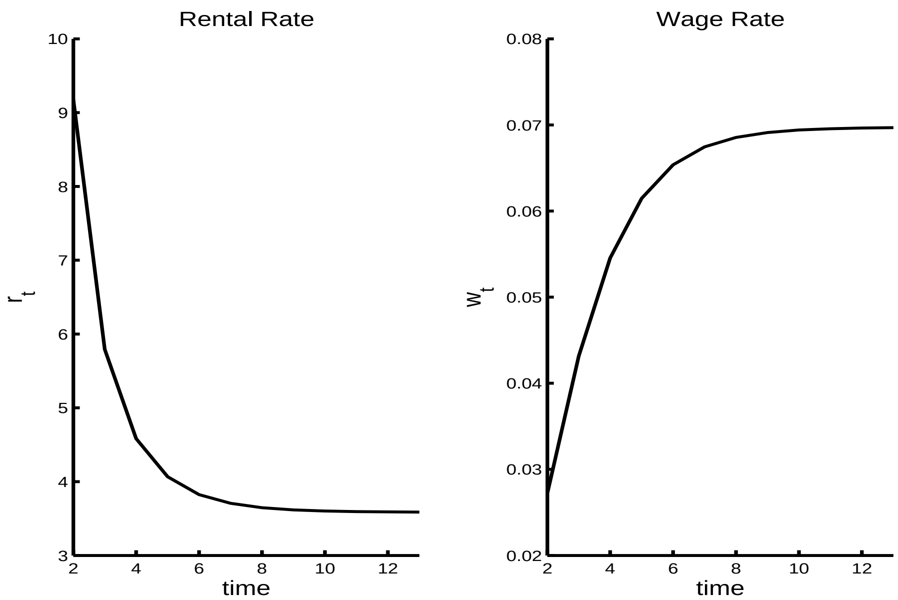

Figure 4.9: Time Paths for the Rental Rate and the Wage Rate

The long-run equilibrium factor prices are given in Table 4.4.

Table 4.4: Equilibrium Factor Prices

r_s	w_s
3.5877	0.0697

Hence, the rental rate is clearly seen to be decreasing over time until converging to its steady state value, whereas the wage rate follows an increasing path over time until stabilizing at its steady state value.

The time paths of the first period consumption are given in Figure 4.10. The consumption of both goods are increasing over time until the steady state is reached. Thus, during the transition to the steady state, each new generation enjoys more of both goods to consume during the first period of their lives. The time paths of second period consumption of both goods are given in Figure 4.10. Similarly to the case with first period consumption, it can be noted that each new generation enjoys a higher consumption of each good during their second period of life, until the steady state is reached. Whether future generations are at least as well off as the current generations depends on the position of the economy while moving to the steady state.

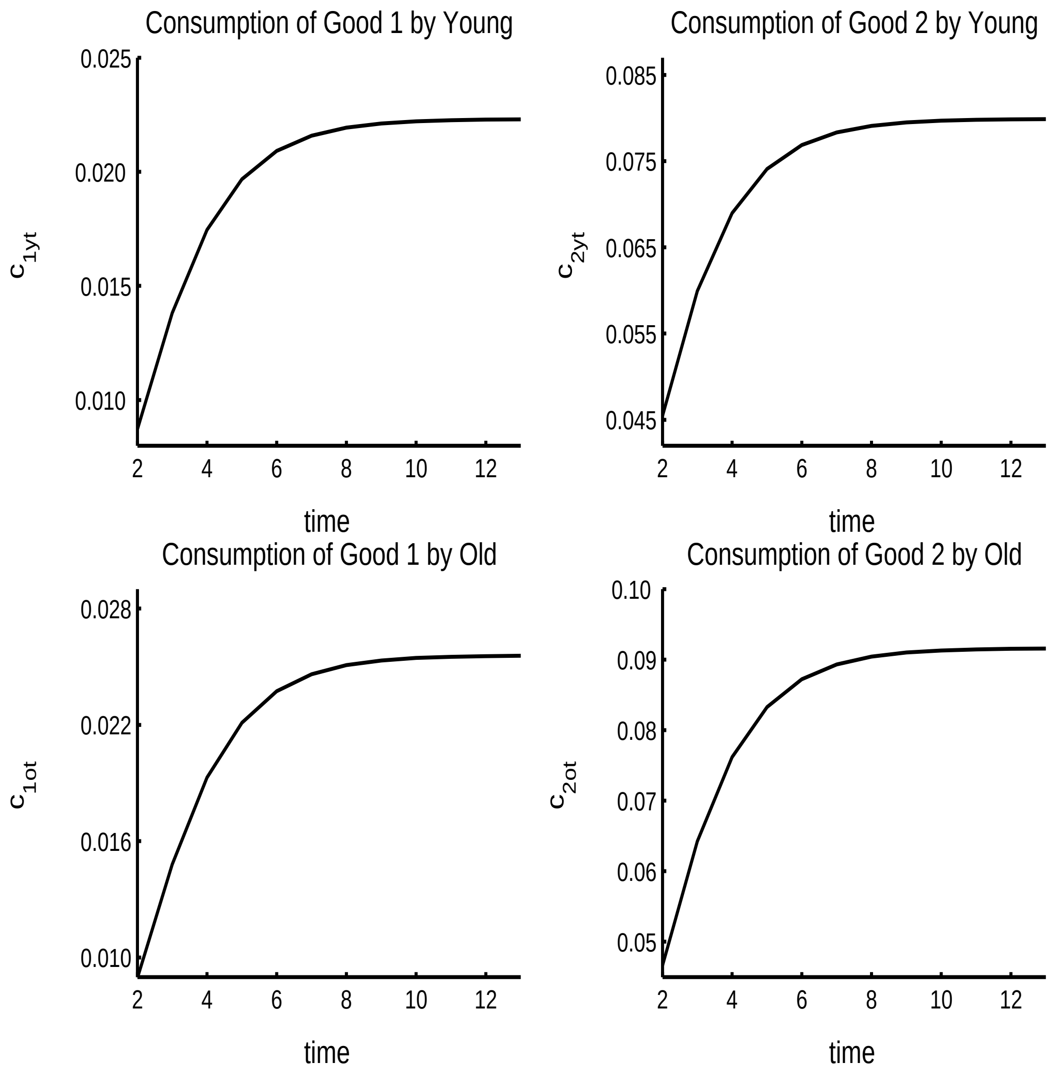

Figure 4.10: Time Paths for Per Capita Consumptions

If the steady state is still far to be reached, then the new generation is strictly better off than the previous generation in terms of the amount of the commodities consumed. If the steady state is already reached, then the future generation will enjoy the same level of consumption as the previous one. This fact is obviously captured by the individual's life time utility too.

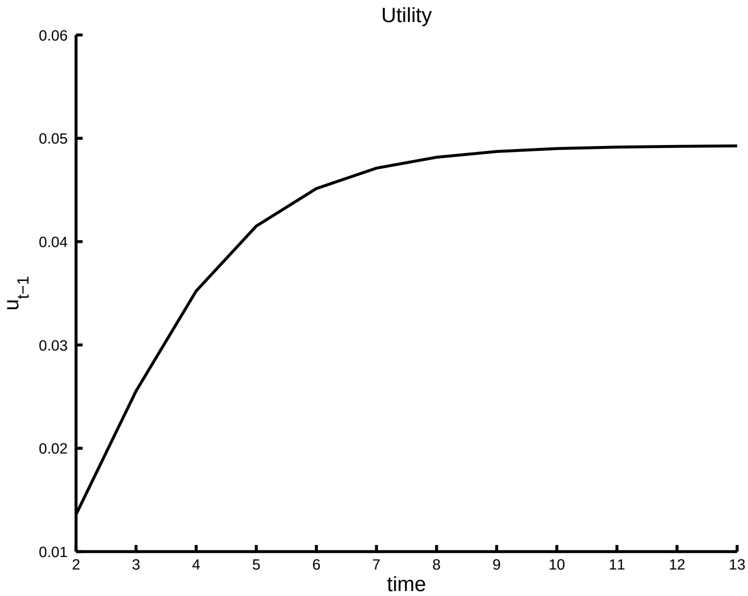

Figure 4.11: Time Path for the Individual's Utility

As it can be seen from Figure 4.11, the time path of the individual's utility is steadily rising over time until the steady state is reached after which all generations face the same welfare level.

Table 4.5: Equilibrium Real Per Capita Consumptions

c_{1ys}	c_{2ys}	c_{1os}	c_{2os}	u_s
0.0223	0.0799	0.0256	0.0916	0.0493

The long-run equilibrium of real per capita consumptions of each good and utility are given in Table 4.5.

In the long-run, per capita magnitudes of all variables would be constant but, the levels of the variables would grow at a constant rate that is equal to population growth rate, n. On the balanced growth path, capital stock as well as total output grow at this natural growth rate of population. Similarly, on the steady state, the total consumption of both goods grow at the population growth rate. The growth rate of total capital stock, total output, and total consumption of each good are plotted in Figure 4.12.

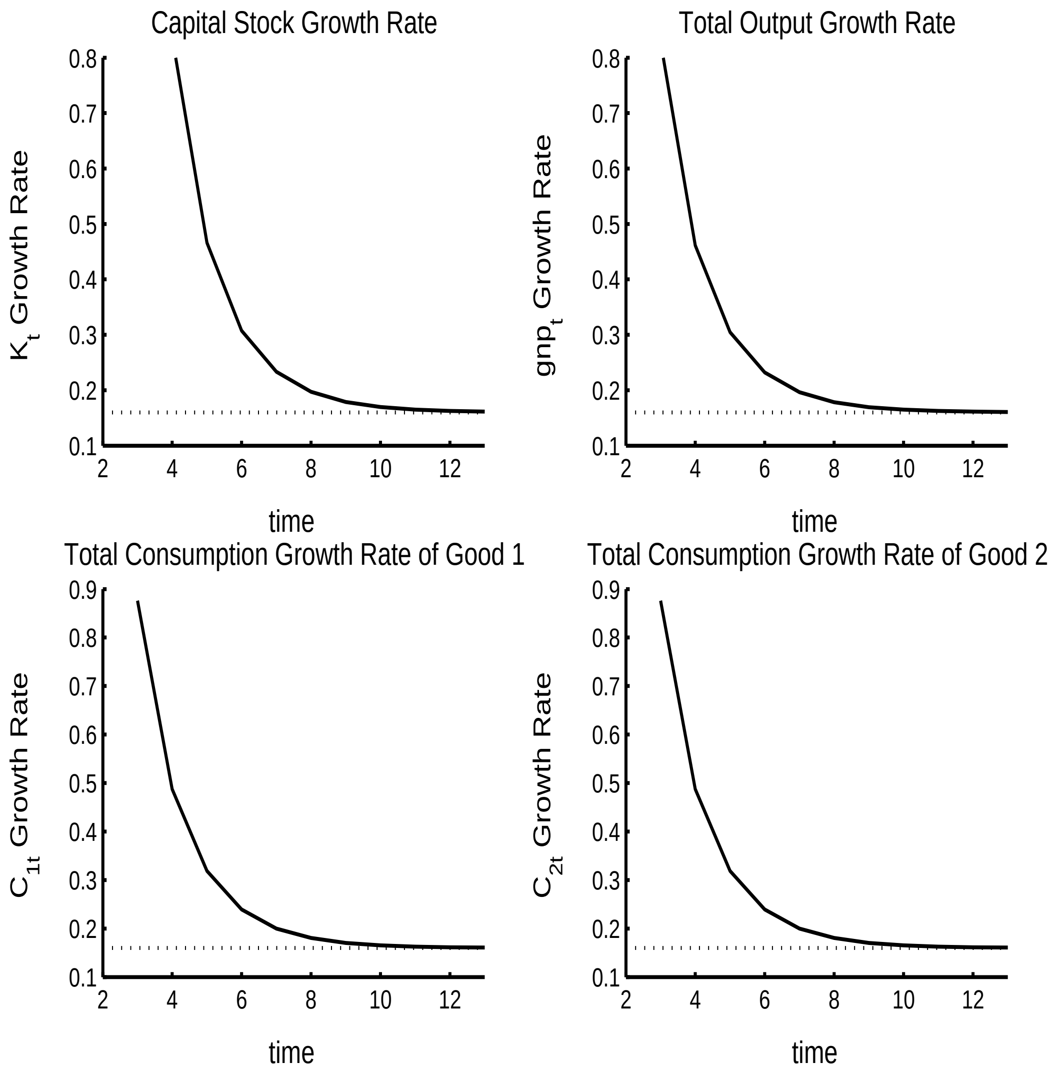

Figure 4.12: Total Capital Stock and Output Growth Rates

It is clearly displayed that all these variables initially grow at a high rate as the economy starts moving from its initial conditions. However, the rate at which the levels of the variables grow, decreases as the economy moves along its transition path until it settles at the natural rate of growth which is equal to the population growth rate.

4.1 The Effect of an Increase in the Population Growth Rate on Long-Run Equilibrium

The effect of population growth rate on $\Delta k_t = 0$ and $\Delta p_t = 0$ loci is plotted in Figure 4.13.

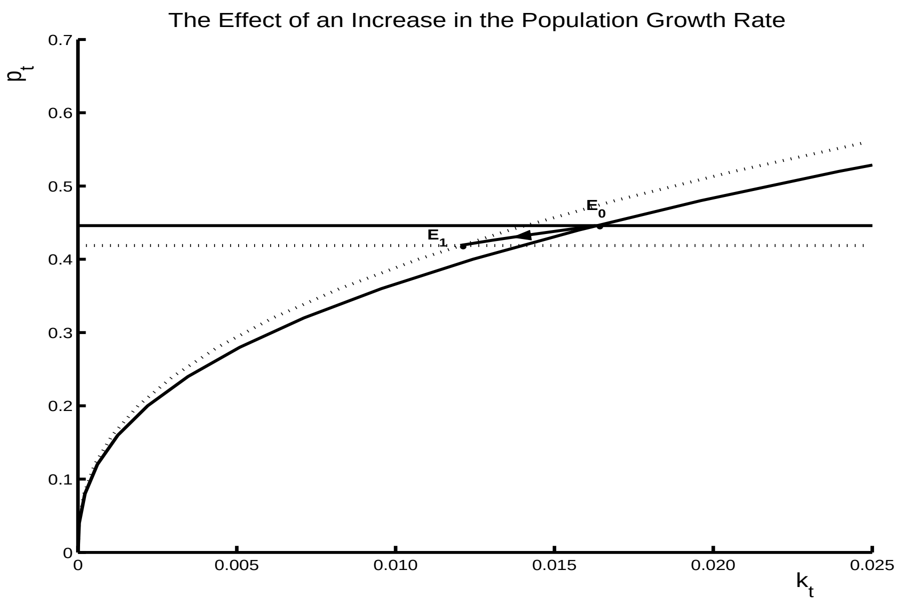

Figure 4.13: The Effects of an Increase in the Population Growth Rate

To create the plots in this figure, we consider first an economy with zero population growth rate that is on its long-run equilibrium, i.e., at point $E-0$. Then, we suppose that that there is a permanent increase in the population growth rate to $n = 0.16$.

Initially, the economy is at point E_0 in Figure 4.13. As a result of the increase in the population growth rate both $\Delta k_t = 0$ and $\Delta p_t = 0$ loci are affected. $\Delta k_t = 0$ locus moves up and the $\Delta p_t = 0$ locus moves down resulting in a lower long-run equilibrium given by point E_1. The path followed by the economy moving from E_0 to E_1 is also plotted in Figure 4.13. The price ratio as well as per capita capital decrease gradually until reaching lower long-run equilibrium values. This establishes that a permanent increase in the population growth rate leads in the long-run to lower equilibrium magnitude of per capita capital and the price ratio.

4.2 The Effect of an Increase in the Saving Rate

The effect of a permanent increase in the saving rate is given in Figure 4.14.

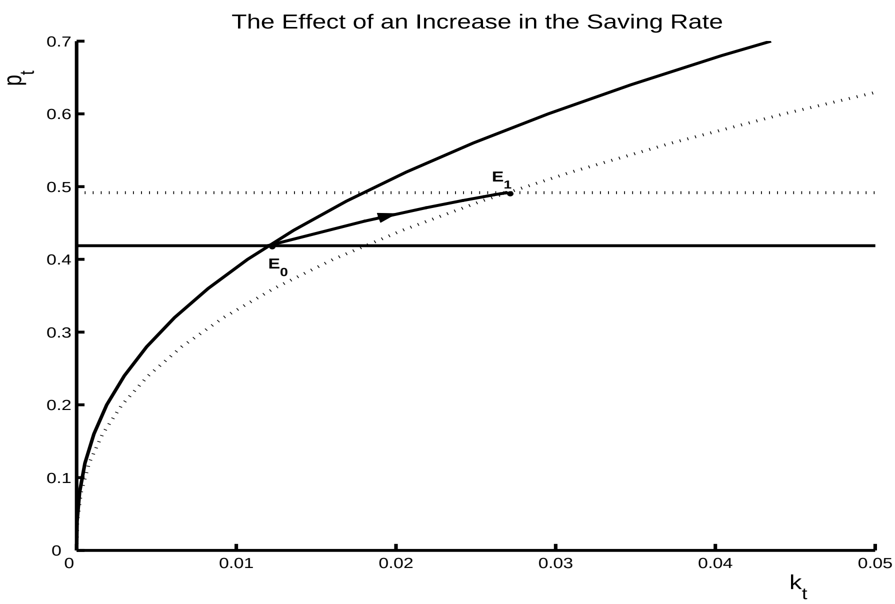

Figure 4.14: The Effects of an Increase in the Saving Rate

First, we consider our positive population growth rate ($n = 0.16$) economy on its balanced growth path, and suppose that there is a permanent increase in the saving rate. That is, a decrease in the parameter μ. Initially, $\mu_0 = 0.8$ (saving rate = 0.2), and suppose that μ_1 becomes equal to 0.7 (saving rate = 0.3).

Initially, the economy is on point E_0 in Figure 4.14. As a result of an increase in the saving rate, both $\Delta k_t = 0$ and $\Delta p_t = 0$ loci are affected.

$\Delta k_t = 0$ locus pivots down and $\Delta p_t = 0$ locus shifts up, resulting in a higher intersection point given by E_1. While our economy's initial endowments and initial price ratio are given by point E_0, at the time of the increase in the saving rate its dynamics are now governed by the new $\Delta k_t = 0$ and $\Delta p_t = 0$ loci. Under these circumstances, k and p rise gradually to their new balanced growth path values given by E_1 at higher than their values on the original balanced growth path given by E_0, implying that a permanent increase in the saving rate leads in the long-run to higher equilibrium magnitude of per capita capital and the price ratio.

CHAPTER V

POPULATION GROWTH RATE DIFFERENCES

This chapter looks into the role that the differences in the population growth rates across nations could play as a determinant of long-run comparative advantages, and discusses the validity of welfare predictions of the static HO model in the long-run.

The effect of population growth rate, n, on steady state magnitudes of key variables is identified, by examining long-run closed form solutions of the autarkic economy. Simulation exercises are also performed to have an idea about the time paths of the model variables for specific model parameter values.

Remembering that the equilibrium steady state price ratio p_s is given by

$$p_s = \Phi^{\frac{\alpha-\beta}{1-\alpha}}, \tag{5.1}$$

where

$$\Phi = \frac{\phi_4 - \phi_3}{\phi_1 + \phi_2},$$

the long-run closed form solutions for the steady state per capita magnitudes are obtained as

$$
\begin{aligned}
k_s &= \phi_4 \Phi^{\frac{\alpha}{1-\alpha}}, && (5.2)\\
w_s &= (1-\alpha)\epsilon^{\alpha}\Phi^{\frac{\alpha}{1-\alpha}}, && (5.3)\\
r_s &= \alpha\epsilon^{\alpha-1}\frac{1}{\Phi}. && (5.4)\\
x_{1s} &= \frac{\delta\bar{l}}{\delta-\epsilon}\epsilon^{\alpha}\Phi^{\frac{\beta}{1-\alpha}} + \frac{1}{\delta-\epsilon}\delta^{\beta}\phi_4\Phi^{\frac{2\alpha-1}{1-\alpha}} && (5.5)\\
x_{2s} &= -\frac{\epsilon\bar{l}}{\delta-\epsilon}\delta^{\beta}\Phi^{\frac{\beta}{1-\alpha}} + \frac{1}{\delta-\epsilon}\delta^{\beta}\phi_4\Phi^{\frac{\alpha+\beta-1}{1-\alpha}} && (5.6)\\
c_{1ys} &= \mu\theta(1-\alpha)\bar{l}\epsilon^{\alpha}\Phi^{\frac{\alpha}{1-\alpha}}, && (5.7)\\
c_{2ys} &= \mu(1-\theta)(1-\alpha)\bar{l}\epsilon^{\alpha}\Phi^{\frac{\beta}{1-\alpha}}, && (5.8)\\
c_{1os} &= 1-\mu)\theta(1-\alpha)\bar{l}\epsilon^{\alpha}(1+\alpha\epsilon^{\alpha-1}\frac{1}{\Phi})\Phi^{\frac{\alpha}{1-\alpha}}, && (5.9)\\
c_{2os} &= (1-\mu)(1-\theta)(1-\alpha)\bar{l}\epsilon^{\alpha}(1+\alpha\epsilon^{\alpha-1}\frac{1}{\Phi})\Phi^{\frac{\beta}{1-\alpha}}. && (5.10)
\end{aligned}
$$

5.1 The Effect of the Population Growth Rate on the Long-run Model Variables

Corollary 1 *The equilibrium price ratio, p_s, is decreasing in the population growth rate n, if the relatively capital-intensive sector is sector 1, and is increasing in the population growth rate, if it is sector 2.*

The effect of the population growth rate n on the steady state price ratio is given by

$$\frac{\partial p_s}{\partial n} = \left(\frac{\alpha-\beta}{1-\alpha}\right)\Phi^{\frac{\alpha-\beta}{1-\alpha}-1}\frac{\partial\Phi}{\partial n}. \qquad (5.11)$$

In order to evaluate the sign of the above expression, we need to find out the sign of $\frac{\partial \Phi}{\partial n}$.

First of all,

$$\begin{aligned}\phi_4 - \phi_3 &= \frac{1}{1+n}(1-\mu)(1-\beta)\bar{l}\left(\delta^\beta - (1-\theta)\beta(\delta-\epsilon)\delta^{\beta-1}\right),\\ \phi_1 + \phi_2 &= (1-\theta)(1-\beta)\bar{l}(\delta-\epsilon)\left(\mu + \frac{1-\mu}{1+n}\right) + \epsilon\bar{l}.\end{aligned}$$

So,

$$\begin{aligned}\Phi = \frac{\phi_4-\phi_3}{\phi_1+\phi_2} &= \frac{(1-\mu)(1-\beta)\bar{l}(\delta^\beta - (1-\theta)\beta(\delta-\epsilon)\delta^{\beta-1})}{(1+n)((1-\theta)(1-\beta)\bar{l}(\delta-\epsilon)(\mu+\frac{1-\mu}{1+n}) + \epsilon\bar{l})}\\ &= \frac{(1-\mu)(1-\beta)\delta^\beta[1-(1-\theta)\beta(1-\frac{\epsilon}{\delta})]}{(1-\theta)(1-\beta)(\delta-\epsilon)(1+n\mu)+(1+n)\epsilon}.\end{aligned}$$

Taking the derivative of the above with respect to n results in

$$\begin{aligned}\frac{\partial \Phi}{\partial n} &= \frac{-(1-\mu)(1-\beta)\delta^\beta[1-(1-\theta)\beta(1-\frac{\epsilon}{\delta})][(1-\theta)(1-\beta)(\delta-\epsilon)\mu+\epsilon]}{[(1-\theta)(1-\beta)(\delta-\epsilon)(1+n\mu)+(1+n)\epsilon]^2}\\ &= \frac{-(1-\mu)(1-\beta)\frac{\delta^\beta}{\epsilon}[1-(1-\theta)\beta(1-\frac{\epsilon}{\delta})][(1-\theta)(1-\beta)(\frac{\delta}{\epsilon}-1)\mu+1]}{[(1-\theta)(1-\beta)(\frac{\delta}{\epsilon}-1)(1+n\mu)+(1+n)]^2}.\end{aligned}$$

Since the denominator of the above expression is positive, we need to find out the sign of the numerator. Now, given the first expression between the squared brackets in the numerator

$$1-(1-\theta)\beta(1-\frac{\epsilon}{\delta}) = 1 - \frac{(1-\theta)(\beta-\alpha)}{(1-\alpha)}.$$

We have

$$\frac{\beta-\alpha}{1-\alpha} < 1,$$

and

$$0 < 1 - \theta < 1.$$

So, multiplying the previous inequality by the above results in

$$\frac{(1-\theta)(\beta-\alpha)}{(1-\alpha)} < (1-\theta) < 1.$$

Thus,

$$0 < 1 - \frac{(1-\theta)(\beta-\alpha)}{1-\alpha}.$$

Similarly, the second expression between the squared brackets in the numerator is

$$(1-\theta)(1-\beta)(\frac{\delta}{\epsilon}-1)\mu + 1 = (1-\theta)(\beta-\alpha)\frac{\mu}{\alpha} + 1. \tag{5.12}$$

Since

$$0 < \beta < 1,$$

and

$$0 < \alpha < 1,$$

we have

$$-\alpha < \beta - \alpha < 1 - \alpha.$$

We also have

$$0 < (1-\theta)\frac{\mu}{\alpha}.$$

So, multiplying the sides of the previous inequality by the above results in

$$-\alpha(1-\theta)\frac{\mu}{\alpha} < (\beta-\alpha)(1-\theta)\frac{\mu}{\alpha}.$$

Thus,

$$-(1-\theta)\mu < (1-\theta)(\beta-\alpha)\frac{\mu}{\alpha}.$$

But since

$$(1-\theta)\mu < 1,$$

we obtain

$$0 < (1-\theta)(\beta-\alpha)\frac{\mu}{\alpha} + 1,$$

establishing that

$$\frac{\partial \Phi}{\partial n} < 0.$$

Now, the sign of $\frac{\partial p_s}{\partial n}$ depends on the sign of $-\left(\frac{\alpha-\beta}{1-\alpha}\right)$.

Therefore,

$$\frac{\partial p_s}{\partial n} \begin{cases} < 0 & \text{for } \alpha > \beta \\ > 0 & \text{for } \alpha < \beta. \end{cases} \tag{5.13}$$

Thus, the equilibrium price of a good decreases with n, if the production of that good is relatively labor-intensive, and increases, if the production of that good is relatively capital-intensive. This implies that countries with a rapidly growing population will have a relative cost advantage in the production of labor-intensive commodities, whereas countries with a slowly growing population will have a relative cost advantage in the production of capital-intensive commodities. In other words, if we start with two countries/regions that are identical in every respect except the population growth rates, the high(low) population growth rate-country will become labor-(capital-)abundant over time, and have a comparative advantage/specialise in the production of labor-(capital) intensive commodity, just as predicted by the HO model.

Corollary 2 *The steady state magnitudes of per capita capital, k_s, and the wage rate, w_s, are decreasing in the population growth rate n, whereas that of the rental rate, r_s, is increasing in the population growth rate n.*

The effect of the population growth rate n on the steady state per capita capital is

$$\frac{\partial k_s}{\partial n} = \Phi^{\frac{\alpha}{1-\alpha}} \frac{\partial \phi_4}{\partial n} + \phi_4 \left(\frac{\alpha}{1-\alpha}\right) \Phi^{\frac{\alpha}{1-\alpha}-1} \frac{\partial \Phi}{\partial n}. \tag{5.14}$$

Since

$$\begin{aligned} \phi_4 &> 0, \\ \frac{\partial \phi_4}{\partial n} = -\frac{\phi_4}{1+n} &< 0, \end{aligned}$$

and since

$$\begin{aligned} \Phi &> 0, \\ \frac{\partial \Phi}{\partial n} &< 0. \end{aligned}$$

Then,

$$\frac{\partial k_s}{\partial n} < 0$$

So, the prediction of the neoclassical economic growth models by Solow (1956), Swan (1956) and in particular the one-sector OLG model by Diamond (1965) concerning the population growth rate is also captured by our two-sector model.

The effect of the population growth rate n on the steady state wage rate is

$$\frac{\partial w_s}{\partial n} = (1-\alpha)\epsilon^{\alpha} \left(\frac{\alpha}{1-\alpha}\right) \Phi^{\frac{\alpha}{1-\alpha}-1} \frac{\partial \Phi}{\partial n} \tag{5.15}$$

Then,

$$\frac{\partial w_s}{\partial n} < 0.$$

Thus, low-population growth countries tend to have a higher wage rate than high-population growth countries, explaining why they would have a comparative disadvantage in the production of labor-intensive commodities. This also implies that unequal population growth rates could induce labor-migration from high- to low-population growth nations in the absence of barriers to labor mobility.

The effect of the population growth rate n on the steady state rental rate is

$$\frac{\partial r_s}{\partial n} = \alpha\epsilon^{\alpha-1}\frac{\partial}{\partial n}\left(\frac{1}{\Phi}\right). \tag{5.16}$$

In order to find out the sign of the above expression, we need to look at the sign of $\frac{\partial}{\partial n}\left(\frac{1}{\Phi}\right)$.

We have

$$\frac{1}{\Phi} = \frac{(1-\theta)(1-\beta)(\delta-\epsilon)(1+n\mu)+(1+n)\epsilon}{(1-\mu)(1-\beta)\delta^{\beta}[1-(1-\theta)\beta(1-\frac{\epsilon}{\delta})]} \tag{5.17}$$

So,

$$\begin{aligned}\frac{\partial}{\partial n}\left(\frac{1}{\Phi}\right) &= \frac{[(1-\theta)(1-\beta)(\delta-\epsilon)\mu+\epsilon](1-\mu)(1-\beta)\delta^{\beta}[1-(1-\theta)\beta(1-\frac{\epsilon}{\delta})]}{[(1-\mu)(1-\beta)\delta^{\beta}(1-(1-\theta)\beta(1-\frac{\epsilon}{\delta}))]^2} \\ &= \frac{\epsilon[(1-\theta)(1-\beta)(\frac{\delta}{\epsilon}-1)\mu+1]}{(1-\mu)(1-\beta)\delta^{\beta}[1-(1-\theta)\beta(1-\frac{\epsilon}{\delta})]}\end{aligned}$$

It was previously shown that

$$(1-\theta)(1-\beta)(\frac{\delta}{\epsilon}-1)\mu+1 = (1-\theta)(\beta-\alpha)\frac{1}{\alpha}\mu+1 > 0$$

and

$$1-(1-\theta)\beta(1-\frac{\epsilon}{\delta}) = 1-\frac{(1-\theta)(\beta-\alpha)}{1-\alpha} > 0.$$

In fact,

$$\frac{\partial}{\partial n}\left(\frac{1}{\Phi}\right) = -\frac{1}{\Phi^2}\frac{\partial\Phi}{\partial n}.$$

Hence,

$$\frac{\partial}{\partial n}\left(\frac{1}{\Phi}\right) > 0.$$

Thus,

$$\frac{\partial r_s}{\partial n} > 0.$$

Hence, countries with a slowly growing population tend to have a lower rental rate on capital than countries with a rapidly growing population. This is what gives these countries a comparative advantage in the production of capital-intensive commodities, and, in the absence of restrictions to capital mobility, would encourage flows of capital from capital-abundant countries to labor-abundant countries. Furthermore, capital flows induced by population aging in one region of the world can transmit the growth and reduce allocation effects of aging globally, as suggested before by Tosun (2003) and Kenç and Sayan (2001).

Corollary 3 *The equilibrium per capita consumptions by youngs of good* 1, c_{1ys}, *and good* 2, c_{2ys}, *are decreasing in the population growth rate* n, *whereas the equilibrium per capita consumptions by the elderly of both goods are ambiguous in the population growth rate,* n.

The first period equilibrium consumptions of both goods decrease in the population growth rate. This inverse relationship between n and equilibrium magnitudes of young generation's consumption follows from the negative relationship between the wage rate and n in the case of good 1, and from the fact that the population growth rate elasticity of the price ratio is higher than the population growth rate elasticity of the wage rate in the case of good 2.

We have

$$\frac{\partial c_{1ys}}{\partial n} = \mu\theta\bar{l}\frac{\partial w_s}{\partial n}.$$

Since

$$\frac{\partial w_s}{\partial n} < 0,$$

$$\frac{\partial c_{1ys}}{\partial n} < 0.$$

As for the first period equilibrium per capita consumption of good 2, we have

$$c_{2ys} = \mu(1-\theta)\bar{l}\frac{w_s}{p_s}.$$

Plugging in the expressions for w_s and p_s in the above gives (5.8), then taking the derivative with respect to the population growth rate n results in

$$\frac{\partial c_{2ys}}{\partial n} = \mu(1-\theta)(1-\alpha)\bar{l}\epsilon^{\alpha}\left(\frac{\beta}{1-\alpha}\right)\Phi^{\frac{\beta}{1-\alpha}-1}\frac{\partial \Phi}{\partial n}, \tag{5.18}$$

Since

$$\frac{\partial \Phi}{\partial n} < 0,$$

$$\frac{\partial c_{2ys}}{\partial n} < 0.$$

Then,

$$\frac{\partial c_{2ys}}{\partial n} = \mu(1-\theta)\bar{l}\frac{1}{p_s^2}\left(p_s\frac{\partial w_s}{\partial n} - w_s\frac{\partial p_s}{\partial n}\right) < 0,$$

implying that

$$\frac{n}{w_s}\frac{\partial w_s}{\partial n} < \frac{n}{p_s}\frac{\partial p_s}{\partial n}.$$

Thus, the population growth rate elasticity of the wage rate is less than the population growth rate elasticity of the price ratio.

The second period equilibrium per capita consumption of good 1 is decreasing in n if $\frac{1}{r_s} > \frac{1-2\alpha}{\alpha}$ and is increasing in n if $\frac{1}{r_s} < \frac{1-2\alpha}{\alpha}$, where r_s is the steady state rental rate.

We have

$$c_{1os} = (1-\mu)\theta\bar{l}w_s(1+r_s).$$

Substituting the long-run closed form solutions of w_s and r_s in the above expression results in

$$c_{1os} = (1+n)\theta\phi_4\Phi^{\frac{\alpha}{1-\alpha}}\left(1+\alpha\epsilon^{\alpha-1}\frac{1}{\Phi}\right).$$

Taking the derivative of the above expression with respect to the population growth rate leads to

$$\begin{aligned}
\frac{\partial c_{1os}}{\partial n} &= \theta\phi_4\Phi^{\frac{\alpha}{1-\alpha}} \\
&+ (1+n)\theta\Phi^{\frac{\alpha}{1-\alpha}}\frac{\partial\phi_4}{\partial n} \\
&+ (1+n)\theta\phi_4\left(\frac{\alpha}{1-\alpha}\right)\Phi^{\frac{\alpha}{1-\alpha}-1}\frac{\partial\Phi}{\partial n} \\
&+ \theta\phi_4\Phi^{\frac{\alpha}{1-\alpha}-1}\alpha\epsilon^{\alpha-1} \\
&+ (1+n)\theta\Phi^{\frac{\alpha}{1-\alpha}-1}\alpha\epsilon^{\alpha-1}\frac{\partial\phi_4}{\partial n} \\
&+ (1+n)\theta\phi_4\alpha\epsilon^{\alpha-1}\left(\frac{\alpha}{1-\alpha}-1\right)\Phi^{\frac{\alpha}{1-\alpha}-2}\frac{\partial\Phi}{\partial n}
\end{aligned}$$

Plugging in

$$\frac{\partial\phi_4}{\partial n} = -\frac{\phi_4}{1+n}$$

in the above expression results in

$$\begin{aligned}
\frac{\partial c_{1os}}{\partial n} &= (1+n)\theta\phi_4\left(\frac{\alpha}{1-\alpha}\right)\Phi^{\frac{\alpha}{1-\alpha}-1}\frac{\partial\Phi}{\partial n} \\
&+ (1+n)\theta\phi_4\alpha\epsilon^{\alpha-1}\left(\frac{\alpha}{1-\alpha}-1\right)\Phi^{\frac{\alpha}{1-\alpha}-2}\frac{\partial\Phi}{\partial n}.
\end{aligned}$$

So, rewriting the above expression we obtain

$$\frac{\partial c_{1os}}{\partial n} = (1+n)\theta\phi_4\left(\frac{1}{1-\alpha}\right)\Phi^{\frac{\alpha}{1-\alpha}-1}\frac{\partial\Phi}{\partial n}(\alpha+(2\alpha-1)r_s),$$

where

$$r_s = \alpha\epsilon^{\alpha-1}\left(\frac{1}{\Phi}\right).$$

Since

$$\frac{\partial\Phi}{\partial n} < 0,$$

$$\frac{\partial c_{1os}}{\partial n}\begin{cases} < 0 & \text{if } \alpha + (2\alpha - 1)r_s > 0 \\ > 0 & \text{if } \alpha + (2\alpha - 1)r_s < 0. \end{cases} \tag{5.19}$$

Alternatively,

$$\frac{\partial c_{1os}}{\partial n}\begin{cases} < 0 & \text{if } \frac{1}{r_s} > \frac{1-2\alpha}{\alpha} \\ > 0 & \text{if } \frac{1}{r_s} < \frac{1-2\alpha}{\alpha}. \end{cases} \tag{5.20}$$

The effect of the population growth rate n on c_{1os} can also be written in terms of the system's parameters as follows:

$$\frac{\partial c_{1os}}{\partial n}\begin{cases} < 0 & \text{if } \Phi > (1 - 2\alpha)\epsilon^{\alpha-1} \\ > 0 & \text{if } \Phi < (1 - 2\alpha)\epsilon^{\alpha-1}. \end{cases} \tag{5.21}$$

Similarly the second period per capita consumption of good 2 is decreasing in n if $\frac{1}{r_s} > \frac{1-(\alpha+\beta)}{\beta}$ and is increasing in n if $\frac{1}{r_s} < \frac{1-(\alpha+\beta)}{\beta}$.

We have

$$c_{2os} = (1-\mu)(1-\theta)\bar{l}\frac{w_s}{p_s}(1+r_s).$$

Substituting the long-run closed form solutions of w_s, p_s and r_s in the above expression results in

$$c_{2os} = (1+n)(1-\theta)\phi_4\Phi^{\frac{\beta}{1-\alpha}}\left(1 + \epsilon^{\alpha-1}\frac{1}{\Phi}\right).$$

Taking the derivative of the above expression with respect to the population growth rate leads to

$$\begin{aligned}
\frac{\partial c_{2os}}{\partial n} &= (1-\theta)\phi_4\Phi^{\frac{\beta}{1-\alpha}} \\
&+ (1+n)(1-\theta)\Phi^{\frac{\beta}{1-\alpha}}\frac{\partial\phi_4}{\partial n} \\
&+ (1+n)(1-\theta)\phi_4\left(\frac{\beta}{1-\alpha}\right)\Phi^{\frac{\beta}{1-\alpha}-1}\frac{\partial\Phi}{\partial n} \\
&+ (1-\theta)\phi_4\Phi^{\frac{\beta}{1-\alpha}-1}\alpha\epsilon^{\alpha-1} \\
&+ (1+n)(1-\theta)\Phi^{\frac{\beta}{1-\alpha}-1}\alpha\epsilon^{\alpha-1}\frac{\partial\phi_4}{\partial n} \\
&+ (1+n)(1-\theta)\phi_4\alpha\epsilon^{\alpha-1}\left(\frac{\beta}{1-\alpha}-1\right)\Phi^{\frac{\beta}{1-\alpha}-2}\frac{\partial\Phi}{\partial n}.
\end{aligned}$$

Plugging in

$$\frac{\partial\phi_4}{\partial n} = -\frac{\phi_4}{1+n}$$

in the above expression results in

$$\begin{aligned}
\frac{\partial c_{2os}}{\partial n} &= (1+n)(1-\theta)\phi_4\left(\frac{\beta}{1-\alpha}\right)\Phi^{\frac{\beta}{1-\alpha}-1}\frac{\partial\Phi}{\partial n} \\
&+ (1+n)(1-\theta)\phi_4\alpha\epsilon^{\alpha-1}\left(\frac{\beta}{1-\alpha}-1\right)\Phi^{\frac{\beta}{1-\alpha}-2}\frac{\partial\Phi}{\partial n}.
\end{aligned}$$

So, rewriting the above expression we obtain

$$\frac{\partial c_{2os}}{\partial n} = (1+n)(1-\theta)\phi_4\left(\frac{1}{1-\alpha}\right)\Phi^{\frac{\beta}{1-\alpha}-1}\frac{\partial\Phi}{\partial n}(\beta+(\beta+\alpha-1)r_s),$$

where

$$r_s = \alpha\epsilon^{\alpha-1}\left(\frac{1}{\Phi}\right).$$

Since

$$\frac{\partial\Phi}{\partial n} < 0,$$

$$\frac{\partial c_{2os}}{\partial n}\begin{cases} <0 & \text{if } \beta+(\beta+\alpha-1)r_s > 0 \\ >0 & \text{if } \beta+(\beta+\alpha-1)r_s < 0. \end{cases} \tag{5.22}$$

Equivalently,

$$\frac{\partial c_{2os}}{\partial n}\begin{cases} < 0 & \text{if } \frac{1}{r_s} > \frac{1-(\alpha+\beta)}{\beta} \\ > 0 & \text{if } \frac{1}{r_s} < \frac{1-(\alpha+\beta)}{\beta}. \end{cases} \tag{5.23}$$

The effect of the population growth rate n on c_{2os} can also be written in terms of the system's parameters as follows,

$$\frac{\partial c_{2os}}{\partial n}\begin{cases} < 0 & \text{if } \Phi > \frac{\alpha}{\beta}(1-\alpha-\beta)\epsilon^{\alpha-1} \\ > 0 & \text{if } \Phi < \frac{\alpha}{\beta}(1-\alpha-\beta)\epsilon^{\alpha-1}. \end{cases} \tag{5.24}$$

This implies that the trade effects on the welfare of the two countries would not be as straightforward to predict as in the case of the static HO model. The welfare results under autarky are driven by the relationship between the values of production parameters and the rental rate r_s which itself varies with n as shown by (5.1). Given that trade will lead to the establishment of a common rental rate between the autarky rentals of trading nations distinguished solely by their population growth rates, the welfare level of each nation may change in either direction, depending on how different consumption variables are affected by the trade-induced change in rentals on capital. Results indicating that trade would not necessarily lead to welfare gains for both countries and might not even be Pareto-superior to autarky have been presented in previous studies based on OLG models with stationary populations (see, for example, Fried (1980), Mountford (1998)).

5.2 Summary

Our discussion of the closed-form solutions to the 2x2 OLG model in this chapter has shown that of the two countries/regions that are identical in every respect except the population growth rates, the high(low) population growth rate-country will become labor-(capital-) abundant over time, and must be expected to have a comparative advantage in the production of labor-(capital-) intensive commodity, as suggested by the static HO model. The welfare effects of trade between two countries, however, were found to depend on the values of system parameters, and hence, were ambiguous, adding unequal population growth rates to the list of previously suggested reasons explaining why trade may not improve welfare for both parties in a dynamic, OLG set-up.

5.3 A Numerical Example

The simulation exercise performed in this section does not only complement the analytical findings in the previous section, but also helps to visualize the time paths of the model variables under study. The model parameters are chosen as follows:

Table 5.1: Model Parameter Values

α	β	μ	θ	l
0.50	0.30	0.80	0.40	1

Two values for the population growth rate n are chosen and each is compared to the case of zero population growth rate ($n = 0$): $n = 0.16$ (roughly the equivalent of a 0.05% annual population growth rate, the unweighted average of the population growth rate for the high income countries between $1980 - 2002$ and the projected values from $2002 - 2015$, WDI (2004)), and $n = 0.7$ (the equivalent of a 1.8% annual population growth rate, the unweighted average of the population growth rate for the low income countries between $1980 - 2002$ and the projected values from $2002 - 2015$, WDI (2004))

The time paths of per capita capital and the price ratio for the different values

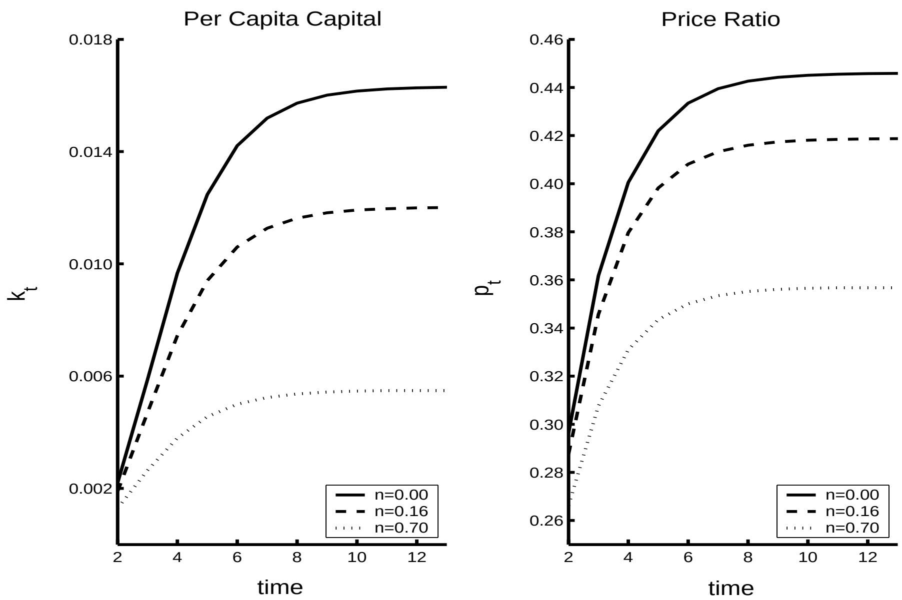

Figure 5.1: Time Paths for Per Capita Capital and the Price Ratio

of the population growth rates are given in Figure 5.1. As it was shown analytically, the long-run equilibrium per capita capital magnitude is decreasing in the population growth rate. The long-run equilibrium price ratio is also decreasing in the population growth rate, since the relatively capital intensive sector is sector 1, or equivalently $\alpha > \beta$.

It must also be noted that the path for per capita capital in the higher population growth rate-country lies below that for the lower population growth rate-country along the transition as well as the long-run equilibrium. This also holds for the path of the price ratio.

The equilibrium magnitudes of per capita capital and the price ratio for this numerical example are given in Table 5.2.

The time paths of the rental rate and the wage rate are given in Figure 5.2. The time path of the rental rate for the higher population growth rate lies above

Table 5.2: Equilibrium of k and p for Given Values of n

n	0	0.16	0.70
k_s	0.0163	0.0120	0.0055
p_s	0.4459	0.4187	0.3568

that for the lower population growth rate. The time path of the wage rate associated with the higher population growth rate lies below that associated with the lower population growth rate. It is clearly seen that the long-run equilibrium rental rate is increasing in the population growth rate, whereas the long-run equilibrium wage rate is decreasing in the population growth rate.

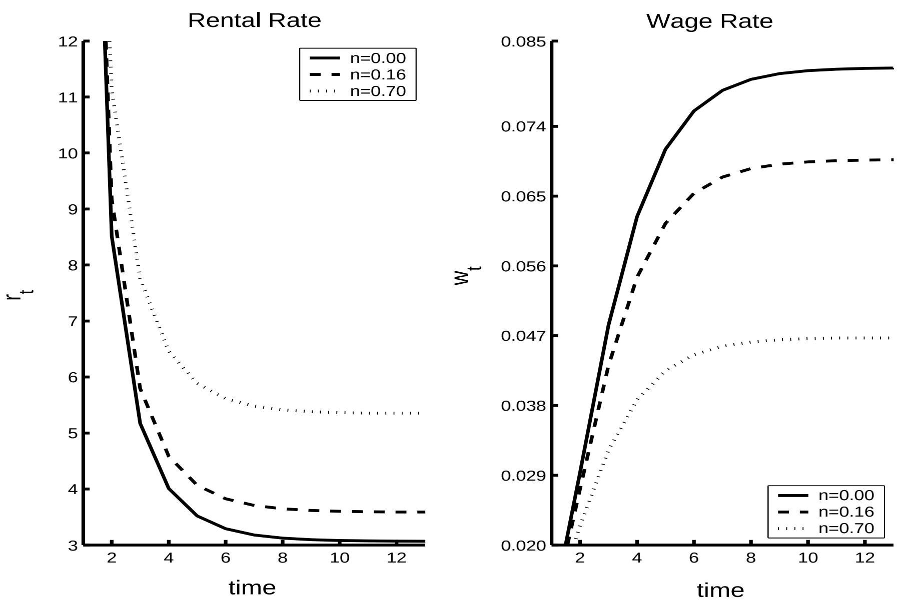

Figure 5.2: Time Paths for the Rental Rate and the Wage Rate

The production sector's long-run equilibrium factor prices are given in Table 5.3.

The time paths of per capita production of both goods are given in Figure 5.3. It is clearly seen that the higher the population growth rate is, the lower the per

Table 5.3: Equilibrium Factor Prices for some Values of n

n	0	0.16	0.70
r_s	3.0656	3.5880	5.3541
w_s	0.0816	0.0697	0.0467

capita production of both goods. In the long-run, per capita production settles on a lower magnitude for the higher population growth rate-country.

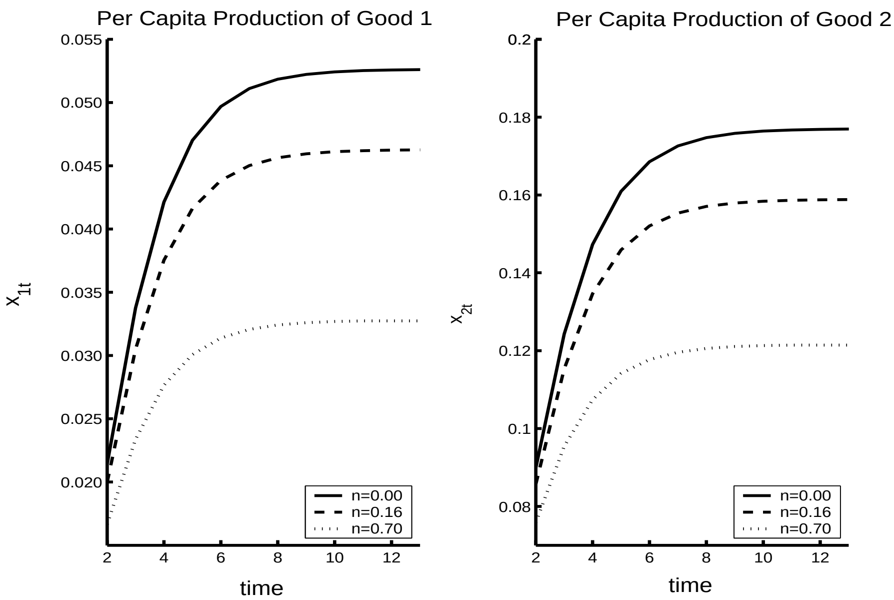

Figure 5.3: Time Paths for Per Capita Production

The time paths of per capita consumption of both goods is given in Figure 5.4. As it was shown analytically, the higher the population growth rate is, the lower the long-run equilibrium magnitude of per capita consumption of both goods by young will be. It is also clear that the path of per capita consumption by young of both goods under the higher population growth rate lies below that under the lower population growth rate. Hence, every new larger generation is worse off when young as it has less to consume from both goods than the previous generation. In particular, if the population growth rate is positive than the new young generation is worse off, otherwise it is better off.

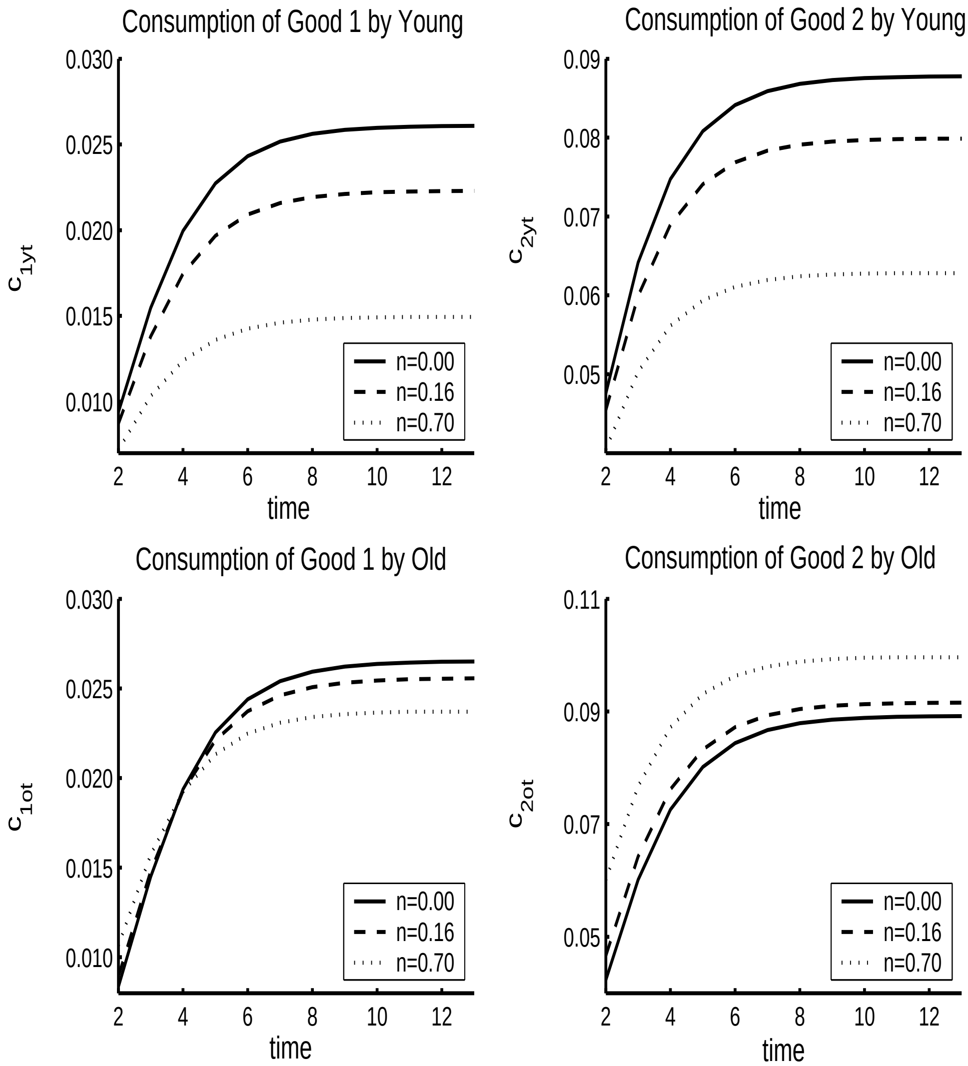

Figure 5.4: Time Paths for Per Capita Consumptions

However, the effect of the population growth rate on per capita consumption by old of both goods does not obey a uniform pattern. In fact, it can be noticed that per capita consumption of good 1 by old corresponding to the high population growth rate is temporarily and slightly higher than that corresponding to the lower population growth rate at the early stages of transition. Later, as the economy moves towards the steady state, it becomes lower and settles on a lower long-run equilibrium.

The consumption side's long-run equilibrium consumption and utility are given in Table 5.4.

Table 5.4: Equlibrium Values of Per Capita Consumptions for some Values of n

n	0	0.16	0.70
c_{1ys}	0.0261	0.0223	0.0149
c_{2ys}	0.0878	0.0799	0.0628
c_{1os}	0.0265	0.0256	0.0237
c_{2os}	0.0892	0.0916	0.0996
u_s	0.0542	0.0493	0.0387

The per capita consumption of good 2 by old though obeys a uniform pattern but with a population growth rate effect opposite in direction to that of per capita consumption by young.

In fact, the time path of per capita consumption of good 2 by old for the high population growth rate lies below that for the low population growth rate. Hence, the higher the population growth rate, the lower the magnitude of the long-run per capita consumption of good 2 by old. So, the overall effect of the population growth rate on per capita consumption by old is ambiguous.

It can be noticed for the case of a positive population growth rate that not only does each new generation enjoys at worse, the same consumption of good 2 during their second period of life compared to the previous generation but also the larger the new generation is, the more per capita consumption of good 2 is enjoyed when old.

So, the higher the population growth rate is, the lower the long-run per capita consumption of both goods when young, the lower the long-run per capita consumption of good 1 when old, but the higher the long-run per capita consumption of good 2 when old. In other words, for high population growth rates the new generation is worse off when young and may be better off when old.

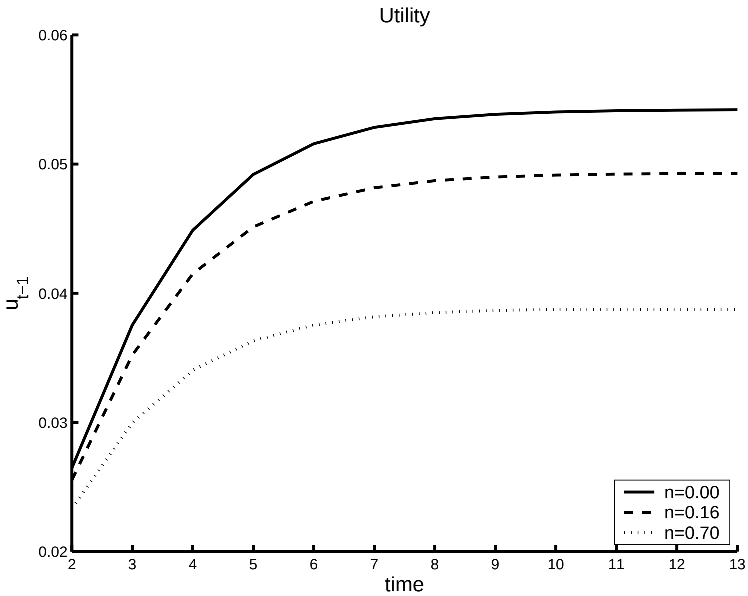

Figure 5.5: Time Paths for the Individual's Utility

The overall effect on welfare is given by the individual's utility as it can be seen from Figure 5.5. It turned out for this case that the higher the population growth rate is the lower the individual's long-run equilibrium utility. Hence, the higher the population growth rate is, the worse off the individual gets.

The behavior of the economy once it has converged to the balanced growth path (k_s, p_s) is characterized by the stationarity of all per capita variables. In other words, capital, output, and consumption in per capita terms become constant, whereas total capital stock, total output, and total consumption grow at the natural rate of growth which is the population growth rate, n.

On the balanced growth path, capital stock as well as total output grow at the natural rate of growth, the population growth rate. This is given by Figure 5.6.

Similarly, on the steady state, the total consumption of good 1 as well as that of good 2 grow at the population growth rate.

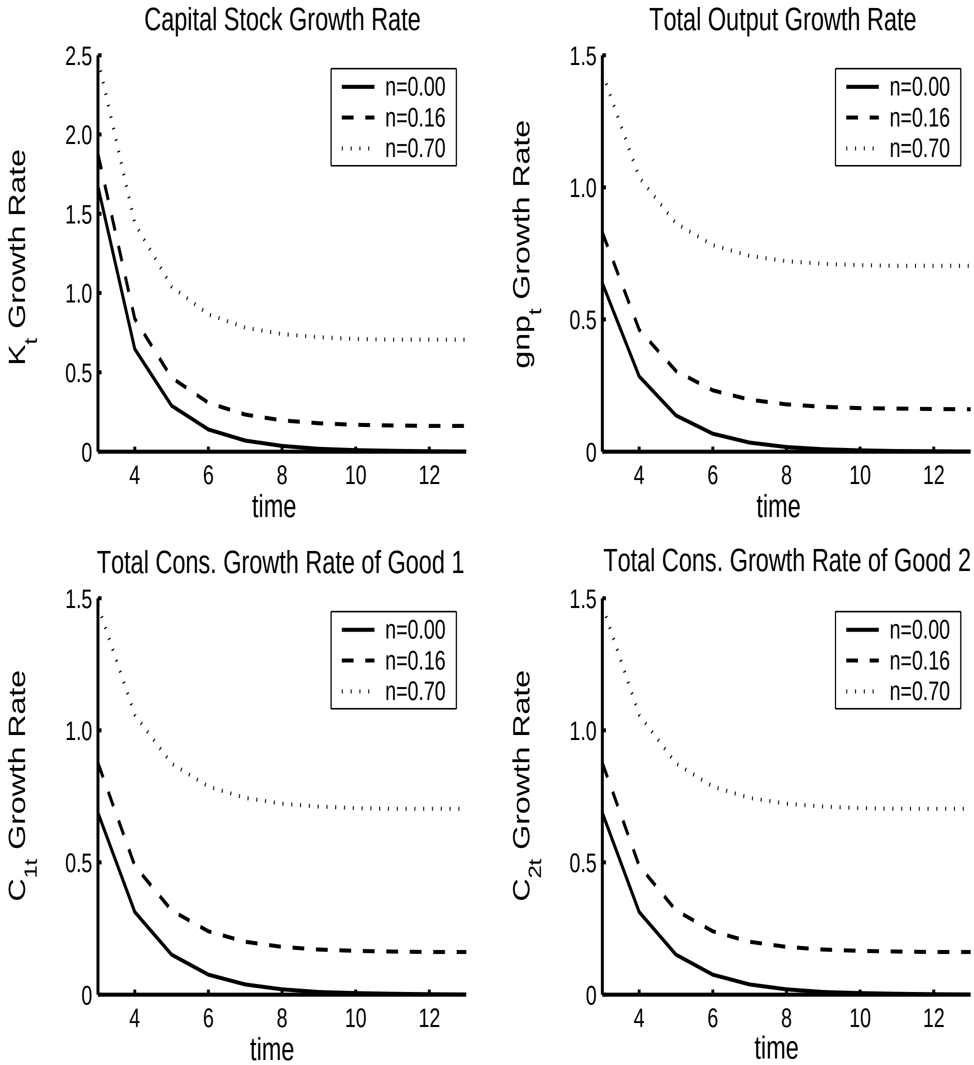

Figure 5.6: Growth Rates of Total Consumption

CHAPTER VI

TRADE BETWEEN EQUAL SIZED COUNTRIES

By the static 2x2x2 Heckscher-Ohlin (HO) model of international trade, differences in relative factor endowments across countries suffice to render trade Pareto-superior to autarky, as long as the factor intensity of production is different for each commodity. While this model has proved to be a very popular starting point for many theoretical and empirical studies, only a few studies in the literature have investigated the validity of predictions of the standard HO model in a dynamic framework. This lack of interest was possibly due to the fact that trade itself would, in the long-run, eliminate the initial differences between relative factor endowments of countries that are assumed to be identical in every other respect, thereby leaving no further incentives for partners to continue trading (Chen (1992)).

This chapter argues that trade may continue to occur in the long-run if there are additional differences to make factor proportions evolve over time, and shows that differential speed of population growth across nations is one of the attributes that could lead to such an evolution in relative factor endowments, allowing for the continuation of trade in the long-run by the criteria set forth in the Heckscher-Ohlin framework. Yet, results in the study also indicate that in the absence of additional differences such as variations in the speed of technological progress or human capital formation, differences in relative endowments of factors will not be sufficient to render trade mutually beneficial in the long-run.

Although changes in relative factor endowments arising due to the differential speed of demographic transition in developing and developed parts of the world are gradually becoming a major factor to affect future patterns of trade, the dynamic trade literature has largely overlooked this issue so far (Sayan (2002)).

This chapter aims to contribute to the literature by extending the static HO model into a dynamic, overlapping generations set-up to look into the role that the differences in the population growth rates across nations could play as a determinant of long-run comparative advantages and to discuss the validity of welfare predictions of the static HO model in the long-run.[3]

For this purpose, we consider a world that is made up of two countries/regions each producing two commodities by using capital and labor. We assume that countries are identical in every other respect than the rates of population growth, and study the implications of this for trade by using the closed-form solutions for the autarky model in Chapter 3 and by solving the trade model analytically. The economies we consider are populated by individuals that live for two periods, and the population in each is allowed to grow constantly at a distinct and exogenously given rate. Such an overlapping generations (OLG) structure capturing the changing savings behavior of individuals over the working and retirement phases of the life cycle implicitly allows the share of savings in national incomes

[3]The results in this chapter are generally consistent with and complement those in Galor and Lin (1997) where the authors establish dynamic micro foundations of the HO model by considering differences in time preference rates across two nations with stationary, rather than growing, populations.

to differ across countries, as differences in the speed of population growth induce variations in relative shares of different age groups in populations. Thus, relative factor endowments evolve, due not only to the changes in labor supply, but also to the changes in capital accumulation resulting from the changing age profiles of populations.

Investigation of the welfare implications of trade in the long-run within this set-up is particularly interesting, as a number of studies based on OLG models with stationary populations have previously suggested that trade would not necessarily lead to mutual welfare gains, and might not even be Pareto-superior to autarky in the long-run (e.g., Fried (1980), Galor (1988*b*) and Galor (1988*a*), Mountford (1998); see also Sayan (2002) for a more detailed discussion).

6.1 Free Trade Scenario

Let L and H denote the low-population growth rate and the high-population growth rate countries, respectively. Let the population growth rate of country L be n^L and that of country L be n^H. Incorporating trade requires that the world-wide supply for both goods is equal to the world-wide demand. Hence, the market clearing condition for good 1 is now given by,

$$\sum_i X_{1t}^i = \sum_i (K_{t+1}^i - K_t^i) + \sum_i (C_{1yt}^i + C_{1ot}^i), \qquad \text{for} \quad i = L, H. \tag{6.1}$$

and the market clearing condition for good 2 is given by,

$$\sum_i X_{2t}^i = \sum_i (C_{2yt}^i + C_{2ot}^i), \qquad \text{for} \quad i = L, H. \tag{6.2}$$

where X_{jt}^i is total output of sector j ($j = 1, 2$) in country i ($i = L, H$); K_t^i is capital stock in country i; C_{jyt}^i is total consumption of good j by the young in country i, and C^i_{jot} is total consumption of good j by the old in country i, all at time t.

Walras' law allows us to focus on the market clearance condition for the consumption good (good 2) alone. Rewriting (6.2) in per capita terms results in

$$N^L_t \bar{x}^L_{2t} + N^H_t \bar{x}^H_{2t} = N^L_t \bar{c}^L_{2yt} + N^L_{t-1} \bar{c}^L_{2ot} + N^H_t \bar{c}^H_{2yt} + N^H_{t-1} \bar{c}^H_{2ot}, \tag{6.3}$$

where the variables with overbars standing for the variables under trade. N^i_t is population size of the young at time t in country i, and N^i_{t-1} is population size of the old at time t in country i. Given that free trade will lead to an equalization of $\bar{p}^i_t$, the price of good 2 in terms of good 1, in both countries in each period,

$$\bar{p}_t = \bar{p}^L_t = \bar{p}^H_t, \tag{6.4}$$

must hold true for every t. Consequently, factor price equalization theorem will hold and result in

$$\begin{aligned} \bar{w}^L_t &= \bar{w}^H_t = \bar{w}_t &(6.5)\\ \bar{r}^L_t &= \bar{r}^H_t = \bar{r}_t &(6.6) \end{aligned}$$

(6.5) and (6.6) follow because

$$\begin{aligned} \bar{w}_t &= (1-\beta)\delta^\beta \bar{p}_t^{\frac{\alpha}{\alpha-\beta}} = (1-\alpha)\epsilon^\alpha \bar{p}_t^{\frac{\alpha}{\alpha-\beta}}, &(6.7)\\ \bar{r}_t &= \beta\delta^{\beta-1} \bar{p}_t^{\frac{\alpha-1}{\alpha-\beta}} = \alpha\epsilon^{\alpha-1} \bar{p}_t^{\frac{\alpha-1}{\alpha-\beta}}, &(6.8) \end{aligned}$$

as previously shown in (3.45) and (3.46). Hence, optimal consumption amounts will be equated across two countries. Focusing on good 2 only, we have

$$\bar{c}^L_{2yt} = \bar{c}^H_{2yt} = \bar{c}_{2yt} = \mu(1-\theta)\bar{l}\frac{\bar{w}_t}{\bar{p}_t} \tag{6.9}$$

$$\bar{c}_{2ot}^L = \bar{c}_{2ot}^H = \bar{c}_{2ot} \quad = \quad (1-\mu)(1-\theta)(1+\bar{r}_t)\bar{l}\frac{\bar{w}_{t-1}}{\bar{p}_t} \tag{6.10}$$

Substituting (6.9), (6.10) in the right hand side (RHS) of (6.3) we get

$$\begin{aligned}(N_t^L + N_t^H)\bar{c}_{2yt} + (N_{t-1}^L + N_{t-1}^H)\bar{c}_{2ot} \quad &= \quad (N_t^L + N_t^H)\mu(1-\theta)\bar{l}\frac{\bar{w}_t}{\bar{p}_t} \\ &+ \quad (N_{t-1}^L + N_{t-1}^H)(1-\mu)(1-\theta)(1+\bar{r}_t)\bar{l}\frac{\bar{w}_{t-1}}{\bar{p}_t}\end{aligned} \tag{6.11}$$

Substituting (6.7) and (6.8) in (6.11) we observe that the RHS of (6.11) equals

$$\begin{aligned}&(N_t^L + N_t^H)\mu(1-\theta)(1-\beta)\bar{l}\delta^\beta \bar{p}_t^{\frac{\beta}{\alpha-\beta}} \\ &+(N_{t-1}^L + N_{t-1}^H)(1-\mu)(1-\theta)(1-\beta)\bar{l}\delta^\beta(1+\beta\delta^{\beta-1}\bar{p}_t^{\frac{\alpha-1}{\alpha-\beta}})\bar{p}_{t-1}^{\frac{\alpha}{\alpha-\beta}}\bar{p}_t^{-\frac{\alpha}{\alpha-\beta}}\end{aligned} \tag{6.12}$$

Remembering from (3.48) that,

$$\bar{x}_{2t}^i = \bar{l}_{2t}^i \delta^\beta \bar{p}_t^{\frac{\beta}{\alpha-\beta}} \qquad \text{for} \qquad i = L, H, \tag{6.13}$$

and substituting this into the left hand side (LHS) of (6.3) we get

$$N_t^L \bar{x}_{2t}^L + N_t^H \bar{x}_{2t}^H = (N_t^L l_{2t}^L + N_t^H l_{2t}^H)\delta^\beta \bar{p}_t^{\frac{\beta}{\alpha-\beta}} \tag{6.14}$$

Given (3.42) and (3.44) showing that

$$\bar{l}_{2t}^i \quad = \quad -\frac{\epsilon \bar{l}}{\delta - \epsilon} + \frac{1}{\delta-\epsilon}\bar{k}_t^i \bar{p}_t^{\frac{1}{\beta-\alpha}}, \tag{6.15}$$

$$\bar{k}_{2t}^i \quad = \quad \frac{\delta}{\delta-\epsilon}\bar{k}_t^i - \frac{\delta\epsilon\bar{l}}{\delta-\epsilon}\bar{p}_t^{\frac{1}{\alpha-\beta}}, \tag{6.16}$$

we can write the following by substituting (6.15) in the RHS of (6.14) by

$$\frac{1}{\delta-\epsilon}\left((\bar{k}_t^L N_t^L + \bar{k}_t^H N_t^H)\bar{p}_t^{\frac{1}{\beta-\alpha}} - \epsilon\bar{l}(N_t^L + N_t^H)\right)\delta^\beta \bar{p}_t^{\frac{\beta}{\alpha-\beta}} \tag{6.17}$$

Now, equating (6.17) and (6.12), we get

$$\begin{aligned}
&\frac{1}{\delta-\epsilon}\left((\bar{k}_t^L N_t^L + \bar{k}_t^H N_t^H)\bar{p}_t^{\frac{1}{\beta-\alpha}} - \epsilon\bar{l}(N_t^L + N_t^H)\right)\delta^\beta \bar{p}_t^{\frac{\beta}{\alpha-\beta}} \qquad (6.18)\\
= \;& (N_t^L + N_t^H)\mu(1-\theta)(1-\beta)\bar{l}\delta^\beta \bar{p}_t^{\frac{\beta}{\alpha-\beta}}\\
&+(N_{t-1}^L + N_{t-1}^H)(1-\mu)(1-\theta)(1-\beta)\bar{l}\delta^\beta(1+\beta\delta^{\beta-1}\bar{p}_t^{\frac{\alpha-1}{\alpha-\beta}})\bar{p}_{t-1}^{\frac{\alpha}{\alpha-\beta}}\bar{p}_t^{-\frac{\alpha}{\alpha-\beta}}
\end{aligned}$$

Rearranging terms,

$$\begin{aligned}
N_t^L\bar{k}_t^L + N_t^H\bar{k}_t^H &= (N_t^L + N_t^H)\bar{l}(\mu(1-\theta)(1-\beta)(\delta-\epsilon)+\epsilon)\bar{p}_t^{\frac{1}{\alpha-\beta}} \qquad (6.19)\\
&+ (N_{t-1}^L + N_{t-1}^H)(1-\mu)(1-\theta)(1-\beta)(\delta-\epsilon)\bar{l}\bar{p}_{t-1}^{\frac{\alpha}{\alpha-\beta}}\bar{p}_t^{\frac{1-\alpha}{\alpha-\beta}}\\
&+ (N_{t-1}^L + N_{t-1}^H)(1-\mu)(1-\theta)(1-\beta)(\delta-\epsilon)\bar{l}\beta\delta^{\beta-1}\bar{p}_{t-1}^{\frac{\alpha}{\alpha-\beta}}
\end{aligned}$$

Let

$$\phi_1 = \mu(1-\theta)(1-\beta)(\delta-\epsilon)\bar{l} + \epsilon\bar{l} \qquad (6.20)$$

$$\bar{\phi}_2 = (1-\mu)(1-\theta)(1-\beta)(\delta-\epsilon)\bar{l} \qquad (6.21)$$

$$\bar{\phi}_3 = (1-\mu)(1-\theta)(1-\beta)(\delta-\epsilon)\bar{l}\beta\delta^{\beta-1} \qquad (6.22)$$

Hence,

$$\begin{aligned}
N_t^L\bar{k}_t^L + N_t^H\bar{k}_t^H &= (N_t^L + N_t^H)\phi_1\bar{p}_t^{\frac{1}{\alpha-\beta}} \qquad (6.23)\\
&+ (N_{t-1}^L + N_{t-1}^H)\bar{\phi}_2\bar{p}_{t-1}^{\frac{\alpha}{\alpha-\beta}}\bar{p}_t^{\frac{1-\alpha}{\alpha-\beta}}\\
&+ (N_{t-1}^L + N_{t-1}^H)\bar{\phi}_3\bar{p}_{t-1}^{\frac{\alpha}{\alpha-\beta}}
\end{aligned}$$

Per capita capital dynamics equation is given by

$$\begin{aligned}
\bar{k}_{t+1}^i &= \frac{1}{1+n^i}(1-\mu)\bar{l}\bar{w}_t\\
&= \frac{1}{1+n^i}(1-\mu)(1-\beta)\bar{l}\delta^\beta\bar{p}_t^{\frac{\alpha}{\alpha-\beta}}
\end{aligned}$$

$$= \frac{1}{1+n^i}\bar{\phi}_4 \bar{p}_t^{\frac{\alpha}{\alpha-\beta}} \qquad \text{for} \qquad i = L, H, \tag{6.24}$$

where

$$\bar{\phi}_4 = (1-\mu)(1-\beta)\bar{l}\delta^{\beta}. \tag{6.25}$$

Writing (6.23) at time $t+1$,

$$\begin{aligned} N_{t+1}^L \bar{k}_{t+1}^L + N_{t+1}^H \bar{k}_{t+1}^H &= (N_{t+1}^L + N_{t+1}^H)\phi_1 \bar{p}_{t+1}^{\frac{1}{\alpha-\beta}} \\ &+ (N_t^L + N_t^H)\bar{\phi}_2 \bar{p}_t^{\frac{\alpha}{\alpha-\beta}} \bar{p}_{t+1}^{\frac{1-\alpha}{\alpha-\beta}} \\ &+ (N_t^L + N_t^H)\bar{\Phi}_3 \bar{p}_t^{\frac{\alpha}{\alpha-\beta}} \end{aligned} \tag{6.26}$$

and substituting (6.24) in (6.26), we get

$$\begin{aligned} \left(\frac{1}{1+n^L}N_{t+1}^L + \frac{1}{1+n^H}N_{t+1}^H\right)\bar{\phi}_4 \bar{p}_t^{\frac{\alpha}{\alpha-\beta}} &= (N_{t+1}^L + N_{t+1}^H)\phi_1 \bar{p}_{t+1}^{\frac{1}{\alpha-\beta}} \\ &+ (N_t^L + N_t^H)\bar{\phi}_2 \bar{p}_t^{\frac{\alpha}{\alpha-\beta}} \bar{p}_{t+1}^{\frac{1-\alpha}{\alpha-\beta}} \\ &+ (N_t^L + N_t^H)\bar{\phi}_3 \bar{p}_t^{\frac{\alpha}{\alpha-\beta}}. \end{aligned} \tag{6.27}$$

Since

$$N_{t+1} = (1+n)N_t,$$

(6.27) becomes

$$\begin{aligned} \left(N_t^L + N_t^H\right)\bar{\phi}_4 \bar{p}_t^{\frac{\alpha}{\alpha-\beta}} &= (N_{t+1}^L + N_{t+1}^H)\phi_1 \bar{p}_{t+1}^{\frac{1}{\alpha-\beta}} \\ &+ (N_t^L + N_t^H)\bar{\phi}_2 \bar{p}_t^{\frac{\alpha}{\alpha-\beta}} \bar{p}_{t+1}^{\frac{1-\alpha}{\alpha-\beta}} \\ &+ (N_t^L + N_t^H)\bar{\phi}_3 \bar{p}_t^{\frac{\alpha}{\alpha-\beta}}. \end{aligned} \tag{6.28}$$

Rearranging terms of (6.28) we can easily get an expression for determining the steady state price ratio:

$$\begin{aligned}
\left(N_t^L + N_t^H\right)(\bar{\phi}_4 - \bar{\phi}_3) &= (N_{t+1}^L + N_{t+1}^H)\phi_1 \left(\frac{\bar{p}_{t+1}^{\frac{1}{\alpha-\beta}}}{\bar{p}_t^{\frac{\alpha}{\alpha-\beta}}}\right)\left(\frac{\bar{p}_{t+1}^{-\frac{\alpha}{\alpha-\beta}}}{\bar{p}_{t+1}^{-\frac{\alpha}{\alpha-\beta}}}\right) \\
&+ (N_t^L + N_t^H)\bar{\phi}_2 \bar{p}_{t+1}^{\frac{1-\alpha}{\alpha-\beta}} \\
&= \left((N_{t+1}^L + N_{t+1}^H)\phi_1 \left(\frac{\bar{p}_{t+1}}{\bar{p}_t}\right)^{\frac{\alpha}{\alpha-\beta}} + (N_t^L + N_t^H)\bar{\phi}_2\right)\bar{p}_{t+1}^{\frac{1-\alpha}{\alpha-\beta}}
\end{aligned}$$

Thus,

$$\bar{p}_{t+1}^{\frac{1-\alpha}{\alpha-\beta}} = \frac{(N_t^L + N_t^H)(\bar{\phi}_4 - \bar{\phi}_3)}{(N_{t+1}^L + N_{t+1}^H)\phi_1 \left(\frac{\bar{p}_{t+1}}{\bar{p}_t}\right)^{\frac{\alpha}{\alpha-\beta}} + (N_t^L + N_t^H)\bar{\phi}_2} \tag{6.29}$$

Therefore,

$$\bar{p}_{t+1} = \left(\frac{(N_{t+1}^L + N_{t+1}^H)\phi_1 \left(\frac{\bar{p}_{t+1}}{\bar{p}_t}\right)^{\frac{\alpha}{\alpha-\beta}} + (N_t^L + N_t^H)\bar{\phi}_2}{(N_t^L + N_t^H)(\bar{\phi}_4 - \bar{\phi}_3)}\right)^{\frac{\beta-\alpha}{1-\alpha}} \tag{6.30}$$

Solving for the steady state equilibrium under trade requires that $\bar{p}_t = \bar{p}_{t+1} = \bar{p}_s$. So, (6.30) implies that there is a $\bar{p}_s > 0$ such that we get

$$\begin{aligned}
\bar{p}_s &= \left(\frac{(N_{s+1}^L + N_{s+1}^H)\phi_1 + (N_s^L + N_s^H)\bar{\phi}_2}{(N_s^L + N_s^H)(\bar{\phi}_4 - \bar{\phi}_3)}\right)^{\frac{\beta-\alpha}{1-\alpha}} \\
&= \left(\frac{\bar{\phi}_2}{\bar{\phi}_4 - \bar{\phi}_3} + \frac{\phi_1}{\bar{\phi}_4 - \bar{\phi}_3}\left(\frac{N_{s+1}^L + N_{s+1}^H}{N_s^L + N_s^H}\right)\right)^{\frac{\beta-\alpha}{1-\alpha}}.
\end{aligned} \tag{6.31}$$

Let

$$\bar{\Phi} = \frac{\bar{\phi}_2}{\bar{\phi}_4 - \bar{\phi}_3} + \frac{\phi_1}{\bar{\phi}_4 - \bar{\phi}_3}\left(\frac{N_{s+1}^L + N_{s+1}^H}{N_s^L + N_s^H}\right). \tag{6.32}$$

Then,

$$\bar{p}_s = \bar{\Phi}^{\frac{\beta-\alpha}{1-\alpha}}. \tag{6.33}$$

Proposition 2 *The common price ratio, $\bar{p}_s$, for this perfect foresight world economy model exists and is unique for all $-1 < n^L < n^H$ and given values of α, β, μ, θ that lie strictly between 0 and 1 such that $\alpha \neq \beta$, and satisfies the interior* *solution condition.*[4]

Proof:

Uniqueness follows from the long-run closed form solution for p_s in (6.31), and the existence is assured by showing that $\bar{\phi}_4 - \bar{\phi}_3 > 0$. An interior solution would require that $\bar{x}^i_{1s} > 0$ and $\bar{x}^i_{2s} > 0$ for $i = L, H$. Now,

$$\begin{aligned} \bar{\phi}_4 - \bar{\phi}_3 &= (1-\mu)(1-\beta)\bar{l}\delta^{\beta} - (1-\mu)(1-\theta)(1-\beta)(\delta-\epsilon)\bar{l}\beta\delta^{\beta-1} \\ &= (1-\mu)(1-\beta)\bar{l}\delta^{\beta}\left(1-(1-\theta)\beta(1-\frac{\epsilon}{\delta})\right) \end{aligned} \tag{6.34}$$

The sign of $\bar{\phi}_4 - \bar{\phi}_3$ depends on the sign of $1-(1-\theta)\beta(1-\frac{\epsilon}{\delta})$. So,

$$1-(1-\theta)\beta(1-\frac{\epsilon}{\delta}) = 1 - \frac{(1-\theta)(\beta-\alpha)}{(1-\alpha)}. \tag{6.35}$$

In fact, $\bar{\phi}_4 - \bar{\phi}_3 > 0$ if and only if $1 - \frac{(1-\theta)(\beta-\alpha)}{(1-\alpha)} > 0$.

Now,

$$\begin{aligned} 1 - \frac{(1-\theta)(\beta-\alpha)}{(1-\alpha)} &> 0 \\ \frac{(1-\theta)(\beta-\alpha)}{(1-\alpha)} &< 1 \\ (1-\alpha)+\alpha(1-\theta) &> (1-\theta)\beta \\ \frac{1-\alpha\theta}{1-\theta} &> \beta \end{aligned}$$

since $\frac{1-\alpha\theta}{1-\theta} > 1$, and $0 < \beta < 1$. Then, $\frac{1-\alpha\theta}{1-\theta} > \beta$ holds for any given α, β and θ.

Thus $\bar{\phi}_4 - \bar{\phi}_3 > 0$ for any given $0 < \alpha < 1$, $0 < \beta < 1$, $0 < \mu < 1$, $0 < \theta < 1$ and $0 < \bar{l}$.

However, for an interior solution, $\bar{x}^i_{js}$ needs to be positive for any $i = L, H$ and

[4] The convergence of the steady state price ratio p_s as $s \to \infty$ is carried out in section 6.3

$j = 1, 2.$

We have

$$\begin{aligned}
\bar{x}^i_{1s} &= \bar{l}^i_{1s}\epsilon^\alpha \bar{p}_s^{\frac{\alpha}{\alpha-\beta}} && (6.36)\\
\bar{x}^i_{2s} &= \bar{l}^i_{2s}\delta^\beta \bar{p}_s^{\frac{\alpha}{\alpha-\beta}} && (6.37)
\end{aligned}$$

Using

$$\begin{aligned}
\bar{l}^i_{1s} &= \frac{\delta}{\delta-\epsilon} - \frac{1}{\delta-\epsilon}\bar{k}^i_s \bar{p}_s^{\frac{1}{\beta-\alpha}}, && (6.38)\\
\bar{l}^i_{2s} &= -\frac{\epsilon}{\delta-\epsilon} + \frac{1}{\delta-\epsilon}\bar{k}^i_s \bar{p}_s^{\frac{1}{\beta-\alpha}}, && (6.39)\\
\frac{\delta}{\delta-\epsilon} &= \frac{\beta(1-\alpha)}{(\beta-\alpha)}, && (6.40)\\
\frac{\epsilon}{\delta-\epsilon} &= \frac{\alpha(1-\beta)}{(\beta-\alpha)}, && (6.41)\\
(1-\beta)\delta^\beta &= (1-\alpha)\epsilon^\alpha, && (6.42)\\
\beta\delta^{\beta-1} &= \alpha\epsilon^{\alpha-1}, && (6.43)\\
\bar{k}^i_s &= \frac{1}{1+n^i}\bar{\phi}_4 \bar{\Phi}^{-\frac{\alpha}{1-\alpha}}, && (6.44)\\
\bar{p}_s &= \bar{\Phi}^{\frac{\beta-\alpha}{1-\alpha}}, && (6.45)
\end{aligned}$$

we get

$$\begin{aligned}
\bar{x}^i_{1s} &= \frac{\beta(1-\alpha)}{(\beta-\alpha)}\epsilon^\alpha \bar{\Phi}^{-\frac{\alpha}{1-\alpha}}\left(1 - \frac{\alpha}{\beta}\frac{(1-\beta)(1-\mu)}{1+n^i}\epsilon^{\alpha-1}\bar{\Phi}\right) && (6.46)\\
\bar{x}^i_{2s} &= \frac{\alpha(1-\alpha)}{(\beta-\alpha)}\epsilon^\alpha \bar{\Phi}^{-\frac{\beta}{1-\alpha}}\left(-1 + \frac{(1-\alpha)(1-\mu)}{1+n^i}\epsilon^{\alpha-1}\bar{\Phi}\right) && (6.47)
\end{aligned}$$

Therefore, $\bar{x}^i_{1s} > 0$ if and only if

$$\begin{cases} \bar{\Phi} > \frac{\beta}{\alpha}\frac{(1+n^i)}{(1-\beta)(1-\mu)}\epsilon^{1-\alpha} & \text{for } \alpha > \beta, \\ \bar{\Phi} < \frac{\beta}{\alpha}\frac{(1+n^i)}{(1-\beta)(1-\mu)}\epsilon^{1-\alpha} & \text{for } \alpha < \beta. \end{cases} \tag{6.48}$$

and $\bar{x}^i_{2s} > 0$ if and only if

$$\begin{cases} \bar{\Phi} < \frac{(1+n^i)}{(1-\alpha)(1-\mu)}\epsilon^{1-\alpha} & \text{for } \alpha > \beta, \\ \bar{\Phi} > \frac{(1+n^i)}{(1-\alpha)(1-\mu)}\epsilon^{1-\alpha} & \text{for } \alpha < \beta. \end{cases} \tag{6.49}$$

∎

6.2 Long-run Closed Form Solutions under Trade

Once $\bar{p}_s$ is determined, the steady state magnitudes of the rest of the macroeconomic variables will be straightforward to obtain. Since,

$$\bar{k}^i_s = \frac{1}{1+n^i}\bar{\phi}_4 \bar{p}_s^{\frac{\alpha}{\alpha-\beta}} \qquad \text{for} \qquad i = H, L. \tag{6.50}$$

the steady state expression of the per capita capital under trade is obtained as

$$\bar{k}^i_s = \frac{1}{1+n^i}\bar{\phi}_4 \bar{\Phi}^{-\frac{\alpha}{1-\alpha}} \qquad \text{for} \qquad i = L, H. \tag{6.51}$$

by substituting (6.31) in (6.50).

As the wage rate is given by

$$\begin{aligned} \bar{w}_s &= (1-\beta)\delta^\beta \bar{p}_s^{\frac{\alpha}{\alpha-\beta}} \\ &= (1-\alpha)\epsilon^\alpha \bar{p}_s^{\frac{\alpha}{\alpha-\beta}}, \end{aligned} \tag{6.52}$$

the steady state expression for the wage rate under trade is found to be

$$\bar{w}_s = (1-\beta)\delta^\beta \bar{\Phi}^{-\frac{\alpha}{1-\alpha}}$$

$$= (1-\alpha)\epsilon^{\alpha}\bar{\Phi}^{-\frac{\alpha}{1-\alpha}} \tag{6.53}$$

by substituting (6.31) in (6.52).

Likewise, given the expression for steady state rental rate:

$$\begin{aligned} \bar{r}_s &= \beta\delta^{\beta-1}\bar{p}_s^{\frac{\alpha-1}{\alpha-\beta}} \\ &= \alpha\epsilon^{\alpha-1}\bar{p}_s^{\frac{\alpha-1}{\alpha-\beta}}, \end{aligned} \tag{6.54}$$

the steady state rental rate under trade is obtained by substituting (6.31) in (6.54) as

$$\begin{aligned} \bar{r}_s &= \beta\delta^{\beta-1}\bar{\Phi} \\ &= \alpha\epsilon^{\alpha-1}\bar{\Phi}. \end{aligned} \tag{6.55}$$

Finally, the long-run real per capita consumptions under trade are given by

$$\bar{c}_{1ys} = \mu\theta(1-\alpha)\epsilon^{\alpha}\bar{\Phi}^{-\frac{\alpha}{1-\alpha}}, \tag{6.56}$$

$$\bar{c}_{2ys} = \mu(1-\theta)(1-\alpha)\epsilon^{\alpha}\bar{\Phi}^{-\frac{\beta}{1-\alpha}}, \tag{6.57}$$

$$\bar{c}_{1os} = (1-\mu)\theta(1-\alpha)\epsilon^{\alpha}(1+\alpha\epsilon^{\alpha-1}\bar{\Phi})\bar{\Phi}^{-\frac{\alpha}{1-\alpha}}, \tag{6.58}$$

$$\bar{c}_{2os} = (1-\mu)(1-\theta)(1-\alpha)\epsilon^{\alpha}(1+\alpha\epsilon^{\alpha-1}\bar{\Phi})\bar{\Phi}^{-\frac{\beta}{1-\alpha}}. \tag{6.59}$$

6.3 The Role of Population Growth Rate under Trade

In order to proceed with our analysis, let's assume without loss of generality that sector 1 is relatively capital intensive and thus sector 2 is relatively labor intensive. Hence $\alpha > \beta$. In line with the static Heckscher-Ohlin framework, our model suggests that the high population growth rate-country (H) must be expected to have a comparative advantage in the production of good 2 and the a low population growth rate-country (L) must be expected to have a comparative advantage in the production of good 1.

In the long-run, $s \to \infty$. Recalling that we consider differences in the population growth rates only, the initial population sizes of both countries are equal (that is $N_1^H = N_1^L$). Hence, rewriting (6.31) we obtain

$$\bar{p}_s = \left(\left(\frac{(1+n^L)^{s+1} + (1+n^H)^{s+1}}{(1+n^L)^s + (1+n^H)^s} \right) \frac{\phi_1}{\bar{\phi}_4 - \bar{\phi}_3} + \frac{\bar{\phi}_2}{\bar{\phi}_4 - \bar{\phi}_3} \right)^{\frac{\beta-\alpha}{1-\alpha}}, \tag{6.60}$$

and

$$\bar{\Phi} = \left(\frac{(1+n^L)^{s+1} + (1+n^H)^{s+1}}{(1+n^L)^s + (1+n^H)^s} \right) \frac{\phi_1}{\bar{\phi}_4 - \bar{\phi}_3} + \frac{\bar{\phi}_2}{\bar{\phi}_4 - \bar{\phi}_3}. \tag{6.61}$$

Hence, the expression of the steady state price ratio will simply be

$$\bar{p}_s = \bar{\Phi}^{\frac{\beta-\alpha}{1-\alpha}}. \tag{6.62}$$

Now, $\bar{p}_s$ will clearly converge as $\bar{\Phi}$ will converge to

$$\lim_{s\to\infty} \bar{\Phi} = (1+n^H)\frac{\phi_1}{\bar{\phi}_4 - \bar{\phi}_3} + \frac{\bar{\phi}_2}{\bar{\phi}_4 - \bar{\phi}_3}. \tag{6.63}$$

To see this, one can express the first term of $\bar{\Phi}$ in (6.61) as

$$1 + \frac{n^L \left(\frac{1+n^L}{1+n^H}\right)^s + n^H}{\left(\frac{1+n^L}{1+n^H}\right)^s + 1}. \tag{6.64}$$

Since

$$\frac{1+n^L}{1+n^H} < 1,$$

this ratio will approach to 0 as $s \to \infty$. Then the whole expression in (6.64) will converge to $(1+n^H)$ yielding (6.63). Hence,

$$\lim_{s\to\infty} \bar{p}_s = \left((1+n^H)\frac{\phi_1}{\bar{\phi}_4-\bar{\phi}_3} + \frac{\bar{\phi}_2}{\bar{\phi}_4-\bar{\phi}_3}\right)^{\frac{\beta-\alpha}{1-\alpha}}, \tag{6.65}$$

implying that the common steady state price ratio, $\bar{p}_s$, will be driven by n^H, the growth rate of faster growing population in H.

In fact, it could be observed that had there been no difference between population growth rates or had n^L been as high as n^H, steady state price ratio in (6.60) would have been the same as autarky relative price ratio in (3.74). Thus, as long as $n^H > n^L$, trade will cause common relative price ratio to be established at the autarky relative price of faster population growth country H in the long-run.

Corollary 4 *Free trade decreases the relative price of labor-intensive commodity* 2 *in the slow-population growth (capital-abundant) country L, towards that of the fast-population growth (labor-abundant) country H.*

First of all, by looking at the expressions of ϕ's given in (3.63),(3.64) and (3.67) and those given in (6.21),(6.22) and (6.25), it is easily seen that

$$\phi_2 = \frac{1}{1+n}\bar{\phi}_2, \tag{6.66}$$

$$\phi_3 = \frac{1}{1+n}\bar{\phi}_3, \tag{6.67}$$

$$\phi_4 = \frac{1}{1+n}\bar{\phi}_4. \tag{6.68}$$

Now, the above corollary can be easily seen by using (6.66), (6.67), (6.68) to rewrite the expression for the steady state relative price under autarky from (3.74) as

$$p_s^i = \left(\frac{\bar{\phi}_4-\bar{\phi}_3}{(1+n^i)\phi_1+\bar{\phi}_2}\right)^{\frac{\alpha-\beta}{1-\alpha}} \qquad \text{for} \quad i = L, H, \tag{6.69}$$

Rewriting the common relative price ratio expression under trade from (6.65) in a similar form yields

$$\bar{p}_s = \left(\frac{\bar{\phi}_4 - \bar{\phi}_3}{(1+n^H)\phi_1 + \bar{\phi}_2} \right)^{\frac{\alpha-\beta}{1-\alpha}}. \tag{6.70}$$

Since $n^H > n^L$ and $\alpha > \beta$, $p_s^L > \bar{p}_s = p_s^H$. Given that $(1+n^H)$ in the denominator of the ratio in (6.70) is nothing but

$$\lim_{s\to\infty} 1 + \frac{n^L \left(\frac{1+n^L}{1+n^H}\right)^s + n^H}{\left(\frac{1+n^L}{1+n^H}\right)^s + 1}, \tag{6.71}$$

convergence of relative price ratio under trade to $\bar{p}_s = p_s^H$ will be quicker, the higher the difference between n^L and n^H is (or the closer the ratio $\frac{1+n^L}{1+n^H}$ is to 0). In other words, the higher the difference between the population growth rates is, the larger the decrease in the relative price of commodity 2 for the low population growth rate-country will be. This is a significant finding, since it implies that for a high enough difference between n^H and n^L, country H will quickly start acting as a large country that is capable of setting the terms of trade close to its autarky relative price ratio.

Proof:

Let's rewrite the expressions of the price ratios under autarky, given in (6.69), into the following

$$p_s^i = (\bar{\phi}_4 - \bar{\phi}_3)^{\frac{\alpha-\beta}{1-\alpha}} \left(\bar{\phi}_2 + \phi_1(1+n^i)\right)^{\frac{\beta-\alpha}{1-\alpha}} \qquad \text{for} \qquad i = L, H, \tag{6.72}$$

and the steady state expression of the price ratio under trade, given in (6.60), as

$$\bar{p}_s = (\bar{\phi}_4 - \bar{\phi}_3)^{\frac{\alpha-\beta}{1-\alpha}} \left(\bar{\phi}_2 + \phi_1 \frac{(1+n^L)^{s+1} + (1+n^H)^{s+1}}{(1+n^L)^s + (1+n^H)^s} \right)^{\frac{\beta-\alpha}{1-\alpha}}. \tag{6.73}$$

Now, in order to compare p_s^i and $\bar{p}_s$, it is sufficient to just compare the second terms within the second pair of parentheses. Considering first the case of $i = L$

in (6.72) and (6.73),

$$(1+n^L) - \frac{(1+n^H)^{s+1} + (1+n^L)^{s+1}}{(1+n^H)^s + (1+n^L)^s} \quad = \quad \frac{(1+n^H)^s(n^L - n^H)}{(1+n^H)^s + (1+n^L)^s}.$$

Since $n^L < n^H$, the RHS of this equation is negative. Then,

$$(1+n^L) < \frac{(1+n^H)^{s+1} + (1+n^L)^{s+1}}{(1+n^H)^s + (1+n^L)^s}$$

Thus,

$$p_s^L < \bar{p}_s.$$

Now,

$$\begin{aligned}
\lim_{s\to\infty} \frac{(1+n^H)^s(n^L - n^H)}{(1+n^H)^s + (1+n^L)^s} &= \lim_{s\to\infty} \frac{1(n^L - n^H)}{1 + \left(\frac{1+n^L}{1+n^H}\right)^s} \\
&= (n^L - n^H).
\end{aligned}$$

So, it is clear from the above that the higher the difference between n^H and n^L, the smaller the ratio $\left(\frac{1+n^L}{1+n^H}\right)$ is and the quicker the ratio $\frac{(n^L - n^H)}{1+\left(\frac{1+n^L}{1+n^H}\right)^s}$ converges to $(n^L - n^H)$. Hence, the higher the difference between the population growth rates, the quicker the price ratio of the low population growth rate-country converges to that of the high population growth rate-country.

Turning to the case of $i = H$ in (6.72) now,

$$(1+n^H) - \frac{(1+n^H)^{s+1} + (1+n^L)^{s+1}}{(1+n^H)^s + (1+n^L)^s} \quad = \quad \frac{(1+n^L)^s(n^H - n^L)}{(1+n^H)^s + (1+n^L)^s}.$$

Again, $n^H > n^L$ implying that the RHS of this equation is positive. Thus,

$$\frac{(1+n^H)^{s+1} + (1+n^L)^{s+1}}{(1+n^H)^s + (1+n^L)^s} < (1+n^H).$$

implying that

$$p_s^L < \bar{p}_s < p_s^H.$$

Similarly,

$$\begin{aligned}\lim_{s\to\infty}\frac{(1+n^L)^s(n^H-n^L)}{(1+n^H)^s+(1+n^L)^s} &= \lim_{s\to\infty}\frac{\left(\frac{1+n^L}{1+n^H}\right)^s(n^H-n^L)}{1+\left(\frac{1+n^L}{1+n^H}\right)^s} \\ &= 0.\end{aligned}$$

Hence, the higher the difference between the population growth rates, the quicker the price ratio under trade for the high population growth rate-country settles on its autarky level. ∎

Corollary 5 *In the long-run, free trade*

- *reduces the steady state magnitude of per capita capital stock in the low population growth rate-country, without affecting it in the high population growth rate-country, and*
- *leads to a decrease in the steady state wage rate, and an increase in the steady state rental rate of the low population growth rate-country towards the steady state factor prices in the high population growth rate-country.*

Using (3.81) and (6.66) through (6.67), the steady state expression for per capita capital under autarky can be rewritten as

$$k_s^i = \frac{\bar{\phi}_4}{1+n^i}\left(\frac{\bar{\phi}_4-\bar{\phi}_3}{(1+n^i)\phi_1+\bar{\phi}_2}\right)^{\frac{\alpha}{1-\alpha}} \qquad \text{for} \quad i = L, H, \tag{6.74}$$

In a similar form, the steady state expression for per capita capital under trade from (6.51) can be rewritten using (6.63) as

$$\bar{k}_s^i = \frac{\bar{\phi}_4}{1+n^i}\left(\frac{\bar{\phi}_4 - \bar{\phi}_3}{(1+n^H)\phi_1 + \bar{\phi}_2}\right)^{\frac{\alpha}{1-\alpha}} \qquad \text{for} \quad i = L, H, \tag{6.75}$$

under trade. Since $n^H > n^L$, $k_s^H < k_s^L$ by (6.74), and $\bar{k}_s^L < k_s^L$. In other words, trade will pull down per capita level of capital stock in L. Equations (6.74) and (6.75) further imply that the higher the difference between population growth rates is, the larger the effect on per capita capital stock of the low-population growth country L will be.

The steady state wage rate expression under autarky from (3.83) can be rewritten by using (6.66) through (6.67) as

$$w_s^i = (1-\alpha)\epsilon^\alpha\left(\frac{\bar{\phi}_4 - \bar{\phi}_3}{(1+n^i)\phi_1 + \bar{\phi}_2}\right)^{\frac{\alpha}{1-\alpha}} \qquad \text{for} \quad i = L, H \tag{6.76}$$

since $n^H > n^L$, $w_s^H < w_s^L$ as established by (6.76). The common wage rate under trade from (6.53), on the other hand, is rewritten by using (6.63) and given by

$$\bar{w}_s = (1-\alpha)\epsilon^\alpha\left(\frac{\bar{\phi}_4 - \bar{\phi}_3}{(1+n^H)\phi_1 + \bar{\phi}_2}\right)^{\frac{\alpha}{1-\alpha}} \tag{6.77}$$

clearly indicating that $w_s^H = \bar{w}_s < w_s^L$. In addition, equation (6.77) implies that the reduction in the autarky wage rate experienced by the slow-population growth country L will be relatively higher for a larger difference between n^H and n^L.

As for the rental rate, its steady state expression under autarky given in (3.85) can be shown to be equivalent to

$$r_s^i = \alpha\epsilon^{\alpha-1}\left(\frac{(1+n^i)\phi_1 + \bar{\phi}_2}{\bar{\phi}_4 - \bar{\phi}_3}\right) \qquad \text{for} \quad i = L, H, \tag{6.78}$$

by using (6.66) through (6.67). The steady state rental rate expression under trade given in (6.55) is now rewritten by using (6.63) as

$$\bar{r}_s = \alpha\epsilon^{\alpha-1}\left(\frac{(1+n^H)\phi_1 + \bar{\phi}_2}{\bar{\phi}_4 - \bar{\phi}_3}\right). \tag{6.79}$$

Since $n^H > n^L$, $r_s^H > r_s^L$. Once again, the common rental rate after trade would settle at $r_s^H = \bar{r}_s > r_s^L$. It is now straightforward to see that the higher the difference in the population growth rates between the trading partners is, the higher the effect of trade on the rental rate of the low population growth rate-country will be.

Thus, our model suggests, in line with the expectations based on the solution of the autarky model as discussed in the previous section, that the high population growth rate-country (H) will have a comparative advantage in the production of labor-intensive consumption good 2, and the low population growth rate-country (L) will have a comparative advantage in the production of capital-intensive good 1 that serves as both an investment good and a consumption good. Furthermore, relative commodity and factor prices under trade will initially lie between corresponding autarky prices just as in the static Heckscher-Ohlin framework, converging to the pre-trade prices for country H in the long-run. This implies that trade creates a tendency for the high population growth rate-country H to pull the magnitudes of all variables towards its own steady state autarky magnitudes in the long-run. In fact, the larger the difference between population growth rates is, the stronger this tendency will get, more quickly enabling country H to behave as the large country that sets the terms of trade.

The challenges that remain now are i) to show that trade may continue to occur in the long-run despite the equalization of these prices under trade, as long as population growth rates are different, and ii) to compare welfare levels across autarky and trade scenarios.

Corollary 6 *The nations considered may continue trading in the long-run as a result of the differences in population growth rates alone, as the initial pattern of comparative advantages are preserved at the steady state.*

To prove Corollary 6, it suffices to show that domestic markets will not clear. Instead, each country will have an excess supply of one commodity (to be exported), and an excess demand for the other (to be satisfied through imports). Below, we show only the long-run expression for the excess supply of good 2 by country H.

Proof:
The steady state output of good 2 by country H is given by

$$\bar{x}_{2s}^{H} = \bar{l}_{2s}^{H} \delta^{\beta} \bar{p}_s^{\frac{\beta}{\alpha-\beta}}. \tag{6.80}$$

Now, using the following

$$\begin{aligned}
\frac{\delta}{\delta-\epsilon} &= \frac{\beta(1-\alpha)}{\beta-\alpha}, \\
\frac{\epsilon}{\delta-\epsilon} &= \frac{\alpha(1-\beta)}{\beta-\alpha} \\
\alpha\epsilon^{\alpha-1} &= \beta\delta^{\beta-1}, \\
(1-\alpha)\epsilon^{\alpha} &= (1-\beta)\delta^{\beta},
\end{aligned}$$

and substituting (6.33), (6.15) and (6.16) evaluated at the steady state for country H, in (6.80) we get

$$\begin{aligned}
x_{2s}^{H} &= -\left(\frac{\epsilon}{\delta-\epsilon}\right)\delta^{\beta}\bar{\Phi}^{-\frac{\beta}{1-\alpha}} + \left(\frac{\delta}{\delta-\epsilon}\right)\frac{(1-\mu)(1-\beta)}{(1+n^H)}\delta^{2\beta-1}\bar{\Phi}^{\frac{1-\alpha-\beta}{1-\alpha}} \\
&= -\left(\frac{\alpha(1-\beta)}{\beta-\alpha}\right)\delta^{\beta}\bar{\Phi}^{-\frac{\beta}{1-\alpha}} + \left(\frac{\beta(1-\alpha)}{\beta-\alpha}\right)\frac{(1-\mu)(1-\beta)}{(1+n^H)}\delta^{2\beta-1}\bar{\Phi}^{\frac{1-\alpha-\beta}{1-\alpha}} \\
&= -\frac{\alpha(1-\alpha)}{\beta-\alpha}\epsilon^{\alpha}\bar{\Phi}^{-\frac{\beta}{1-\alpha}} + \frac{\alpha(1-\alpha)}{\beta-\alpha}\frac{(1-\mu)(1-\beta)}{(1+n^H)}\epsilon^{2\beta-1}\bar{\Phi}^{\frac{1-\alpha-\beta}{1-\alpha}}
\end{aligned}$$

$$= \frac{\alpha}{\beta-\alpha}(1-\alpha)\epsilon^{\alpha}\bar{\Phi}^{-\frac{\beta}{1-\alpha}}\left(-1+\frac{(1-\mu)(1-\alpha)}{1+n^H}\epsilon^{\alpha-1}\bar{\Phi}\right). \quad (6.81)$$

The long-run total real consumption, on the other hand, is given by

$$\begin{aligned} N_s^H\bar{c}_{2ys}^H + N_{s-1}^H\bar{c}_{2os}^H &= N_s^H\mu(1-\theta)(1-\alpha)\epsilon^{\alpha}\bar{\Phi}^{-\frac{\beta}{1-\alpha}} \\ &+ N_{s-1}^H(1-\mu)(1-\theta)(1-\alpha)\epsilon^{\alpha}(1+\alpha\epsilon^{\alpha-1}\bar{\Phi})\bar{\Phi}^{-\frac{\beta}{1-\alpha}} \\ &= (1-\theta)(1-\alpha)\epsilon^{\alpha}\bar{\Phi}^{-\frac{\beta}{1-\alpha}}N_s^H\left(\mu+\frac{1-\mu}{1+n^H}+\frac{1-\mu}{1+n^H}\alpha\epsilon^{\alpha-1}\bar{\Phi}\right). \end{aligned}$$

Now, excess supply of good 2 by country H (Exs_2^H) is total production of good 2 less total domestic consumption of that good. Thus

$$Exs_2^H = N_s^H\bar{x}_{2s}^H - (N_s^H\bar{c}_{2ys}^H + N_{s-1}^H\bar{c}_{2os}^H).$$

Hence,

$$\begin{aligned} Exs_2^H = (1-\alpha)\epsilon^{\alpha}\bar{\Phi}^{-\frac{\beta}{1-\alpha}}N_s^H\Bigg\{&-\frac{\alpha}{\beta-\alpha}-(1-\theta)\left(\mu+\frac{1-\mu}{1+n^H}\right) \\ &+ \frac{1-\mu}{1+n^H}\alpha\epsilon^{\alpha-1}\bar{\Phi}\left(\frac{1-\alpha}{\beta-\alpha}\right)\Bigg\} \end{aligned}$$

Thus, $Exs_2^H > 0$ if

$$-\frac{\alpha}{\beta-\alpha}-(1-\theta)\left(\mu+\frac{1-\mu}{1+n^H}\right)+\frac{1-\mu}{1+n^H}\alpha\epsilon^{\alpha-1}\bar{\Phi}\left(\frac{1-\alpha}{\beta-\alpha}\right) > 0. \quad (6.82)$$

Claim 1

$$\bar{\Phi} = \frac{1}{\alpha\epsilon^{\alpha-1}[(1-\alpha)-(1-\theta)(\beta-\alpha)]}\left\{\bar{N}\frac{\mu(1-\theta)(\beta-\alpha)+\alpha}{(1-\mu)}+(1-\theta)(\beta-\alpha)\right\},$$

where

$$\bar{N} = \frac{N_{s+1}^L+N_{s+1}^H}{N_s^L+N_s^H}$$

$$= \frac{(1+n^L)N_s^L + (1+n^H)N_s^H}{N_s^L + N_s^H}.$$

Proof:

Substituting (6.21), (6.22), (6.23) and (6.25) in the expression of $\bar{\Phi}$ given in (6.32) we get

$$\begin{aligned}
\bar{\Phi} &= \bar{N}\frac{\mu(1-\theta)(1-\beta)(\delta-\epsilon)+\epsilon}{(1-\mu)(1-\beta)\delta^{\beta}(1-(1-\theta)\beta(1-\frac{\epsilon}{\delta})} + \frac{(1-\theta)(\delta-\epsilon)}{\delta^{\beta}(1-(1-\theta)\beta(1-\frac{\epsilon}{\delta})} \\
&= \bar{N}\frac{\mu(1-\theta)(1-\beta)(\delta-\epsilon)+\frac{\epsilon}{\delta}}{(1-\mu)(1-\beta)\delta^{\beta-1}(1-(1-\theta)\beta(1-\frac{\epsilon}{\delta})} + \frac{(1-\theta)(1-\frac{\epsilon}{\delta})}{\delta^{\beta-1}(1-(1-\theta)\beta(1-\frac{\epsilon}{\delta})}
\end{aligned}$$

∎

Substituting this expression for $\bar{\Phi}$ into the inequality in (6.82) and rearranging terms result in $Exs_2^H > 0$ if

$$\frac{1-\mu}{(1+n^H)(\beta-\alpha)}\left(\bar{N}\frac{\mu(1-\theta)(\beta-\alpha)+\alpha}{(1-\mu)} + (1-\theta)(\beta-\alpha)\right) > \frac{\alpha}{\beta-\alpha} + (1-\theta)\mu + \frac{(1-\mu)(1-\theta)}{1+n^H}.$$

Thus, $Exs_2^H > 0$ if

$$\left(\frac{\bar{N}}{1+n^H} - 1\right)\left(\mu(1-\theta) + \frac{\alpha}{\beta-\alpha}\right) > 0.$$

Since

$$\frac{\bar{N}}{1+n^H} - 1 < 0,$$

$Exs_2^H > 0$ requires that

$$\mu(1-\theta) + \frac{\alpha}{\beta-\alpha} < 0$$

holds. But

$$\mu(1-\theta) < \frac{\alpha}{\alpha-\beta}$$

is always true for $\alpha > \beta$, since $\mu(1-\theta) < 1$ and $\frac{\alpha}{\alpha-\beta} > 1$.

Therefore,

$$Exs_2^H \begin{cases} > 0 & \text{for } \alpha > \beta \\ < 0 & \text{for } \alpha < \beta. \end{cases} \tag{6.83}$$

∎

This implies that the country with a fast growing population (i.e., the labor-abundant-country) will export the labor-intensive commodity 2, as expected.

Having shown that trade may continue to occur at the steady state, we can now compare welfare levels across autarky and trade. Again, as expected from solutions under autarky, it is not obvious that welfare results are consistent with the static HO model.

Corollary 7 *Free trade leads in the long-run to*

- *a decrease in per capita consumption by youngs of both goods in the low population growth rate-country L,*
- *an ambiguous effect on the per capita consumption by olds of both goods in the low population growth rate-country L, but*
- *no change in per capita consumption by youngs and olds of both goods in the high population growth rate-country H.*

The long-run equilibrium real per capita consumption of good 1 by the youngs is given by (3.89) under autarky, and by (6.56) under trade. Since $w_s^H = \bar{w}_s < w_s^L$,

$$c_{1ys}^H = \bar{c}_{1ys} < c_{1ys}^L$$

and a similar ranking can be made for good 2. The long-run equilibrium real per capita consumption by the youngs is given by (3.91) under autarky, and by (6.57) under trade. Since,

$$\frac{w_s^H}{p_s^H} = \frac{\bar{w}_s}{\bar{p}_s} < \frac{w_s^L}{p_s^L},$$
$$c_{2ys}^F = \bar{c}_{2ys} < c_{2ys}^L.$$

Thus, trade leads to a decrease in the youngs' consumption of both commodities in country L in per capita terms.

Such a ranking, however, is not easy to find in the case of per capita consumption of goods by the olds. For commodity 1, per capita consumption by the olds under autarky is given by (3.93) Remembering that trade leads to a decrease in the long-run wage rate but an increase in the long-run rental rate for the slow-population growth country, the overall effect of trade on c_{1os}^L depends on which effect dominates: the trade-induced increase in the rental rate or the trade-induced decrease in the wage rate for the slow population growth rate-country.

Similarly, since per capita consumption of good 2 by the olds under autarky is given by (3.95), the overall effect of trade on c_{2os}^L would also depend on the relative magnitudes of the trade-induced increase in the rental rate or the trade-induced decrease in the wage rate.

To better visualize this, we summarize the pre-trade and post-trade relationships between these two economies' prices and evaluate the rental rate elasticity of welfare.

It has been established before that

$$p_s^L < p_s^H,$$
$$w_s^L < w_s^H, \tag{6.84}$$

$$r_s^L > r_s^H$$

under autarky. Now, when two countries begin trading with each other to make use of these differences in autarky relative prices, they will export the commodity in which they have a relative comparative advantage. This will lead to the establishment of common relative commodity and factor prices lying between respective autarky prices as predicted by the HO model. Thus, the opening of trade will lead to an increase (no change) in p_s^L and w_s^L (p_s^H and w_s^H) and a decrease (no change) in r_s^L (r_s^H). In other words,

$$\begin{aligned} p_s^L &< \bar{p}_s = p_s^H, \\ w_s^L &< \bar{w}_s = w_s^H \\ r_s^L &> \bar{r}_s = r_s^H \end{aligned} \tag{6.85}$$

It is obvious that the long-run welfare of the high population growth rate-country is unaffected by trade. The question now is whether the trade-induced changes in these autarky prices will unambiguously improve the welfare for the other trading party, the slow population growth rate-country, as suggested by the static HO model. The following corollary answers this question.

Corollary 8 *The direction in which the life time utility of an individual in country L will change with the opening of trade depends on the sign of the elasticity of*

$$u_s^L = ({c_{1ys}^L}^{\theta} {c_{2ys}^L}^{1-\theta})^{\mu} ({c_{1os}^L}^{\theta} {c_{2os}^L}^{1-\theta})^{1-\mu} \tag{6.86}$$

with respect to the trade-induced changes in relative commodity or factor prices.

Proof:

Let's consider the reaction of steady state autarky of life time utility of an individual in country L to a trade-induced change in the rental for capital. We consider the trade-induced change in r_s^L only on account of the relative simplicity of expressions. The same result will hold for p_s^L and w_s^L as well but showing this will require more tedious algebraic derivations. Given u_s^L above, the relevant elasticity can be expressed as

$$\begin{aligned} e_{u_s^L, r_s^L} = \quad & \mu\theta e_{c_{1ys}^L, r_s^L} \\ & +\mu(1-\theta) e_{c_{2ys}^L, r_s^L} \\ & +(1-\mu)\theta e_{c_{1os}^L, r_s^L} \\ & +(1-\mu)(1-\theta) e_{c_{2os}^L, r_s^L}, \end{aligned} \tag{6.87}$$

where each e term with a subscript shows the elasticity of the steady state magnitude of the variable denoted by the first term in the subscript with respect to the steady state rental for capital, r_s^L.

Now, using (3.85) and (3.89), (3.91), (3.93) and (3.95) to obtain the steady state magnitudes of per capita consumptions in terms of r_s^L, we get

$$c_{1ys} = \mu\theta(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} r_s^{\frac{\alpha}{\alpha-1}} \tag{6.88}$$

$$c_{2ys} = \mu(1-\theta)(1-\alpha)\epsilon^{\alpha-\beta}\alpha^{\frac{\beta}{1-\alpha}} r_s^{\frac{\beta}{\alpha-1}} \tag{6.89}$$

$$c_{1os} = (1-\mu)\theta(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(r_s^{\frac{\alpha}{\alpha-1}} + r_s^{\frac{\alpha}{\alpha-1}+1}\right) \tag{6.90}$$

$$c_{2os} = (1-\mu)(1-\theta)(1-\alpha)\epsilon^{\alpha-\beta}\alpha^{\frac{\beta}{1-\alpha}}\left(r_s^{\frac{\beta}{\alpha-1}} + r_s^{\frac{\beta}{\alpha-1}+1}\right) \tag{6.91}$$

and evaluating their elasticities with respect to r_s^L we get

$$e_{c_{1ys}^L, r_s^L} = \frac{\alpha}{\alpha-1}, \tag{6.92}$$

$$e_{c_{2ys}^L, r_s^L} = \frac{\beta}{\alpha-1}, \tag{6.93}$$

$$e_{c_{1os}^L, r_s^L} = \frac{\alpha}{\alpha-1} + \frac{r_s^L}{1+r_s^L}, \tag{6.94}$$

$$e_{c^L_{2os},r^L_s} \quad = \quad \frac{\beta}{\alpha-1}+\frac{r^L_s}{1+r^L_s}. \tag{6.95}$$

Substituting (6.92) through (6.95) in (6.87) we get

$$e_{u^L_s,r^L_s} = \theta\left(\frac{\alpha}{\alpha-1}\right)+(1-\theta)\left(\frac{\beta}{\alpha-1}\right)+(1-\mu)\left(\frac{r^L_s}{1+r^L_s}\right). \tag{6.96}$$

But

$$e_{u^L_s,r^L_s}\begin{cases} <0 & \text{if } \frac{1}{r^L_s} > \frac{(1-\mu)(1-\alpha)}{\theta\alpha+(1-\theta)\beta}-1 \\ >0 & \text{otherwise.}\end{cases} \tag{6.97}$$

∎

The autarky steady state rental rate r^L_s will decrease in L after opening of trade as shown in inequality (6.85). Since the elasticity in (6.96) is ambiguous in sign, however, the direction of trade-induced change in u^L_s will not be as straightforward to tell as in the static HO model. This result is consistent with previously cited OLG-GE studies based on stationary populations, as well as the recent work by Sayan (2005) who considers a dynamic OLG-GE extension of the HO model with population growth differentials.

It is conceivable that $e_{u_s,r_s} < 0$ may hold, unless there are additional restrictions on the values of parameters. Therefore, a country with a low population growth rate may very well face a reduction in its autarky level of welfare after beginning to trade with a high population growth rate-country.

6.4 A Numerical Example

In this section several simulation exercises are performed to visualize the time paths of the model variables, as they are analytically challenging to obtain. The model parameters chosen are given in Table 6.1.

Table 6.1: Trade Model Parameter Values

α	β	μ	θ	l
0.50	0.30	0.80	0.40	1

The two trading countries are similar in every respect except for the population growth rate. Hence, for the low population growth rate-country $n = 0.16$ (the equivalent of a 0.05% annual population growth rate, the unweighted average of the population growth rate for the high income countries between 1980-2002 and the projected values from 2002-2015, WDI (2004)), and for the high population growth rate-country, $n = 0.7$ (the equivalent of a 1.8% annual population growth rate, the unweighted average of the population growth rate for the low income countries between 1980-2002 and the projected values from 2002-2015, WDI (2004)).

The time paths of per capita capital and the price ratio for the autarky and the trade scenarios are given in Figure 6.1.

Figure 6.1 shows that the path of per capita capital for the low population growth rate-country under autarky lies above that under trade. This follows from (6.50) establishing that per capita capital and the price ratio are positively related when $\alpha > \beta$. The reduction in the price of good 2 in terms of good 1 for country L as a consequence of trade leads to capital dilution. For the same reason the path of per capita capital for the fast population growth rate-country under trade is above that under autarky. Moreover, the low population growth rate-country's paths of per capita capital lie higher than those of the high population growth rate-country under both scenarios. Thus, in the long-run free trade substantially lowers the magnitude of per capita capital for the low population growth rate-country, while converging to that of the high population growth rate-country.

Consistently with the Heckscher-Ohlin framework, opening of trade between

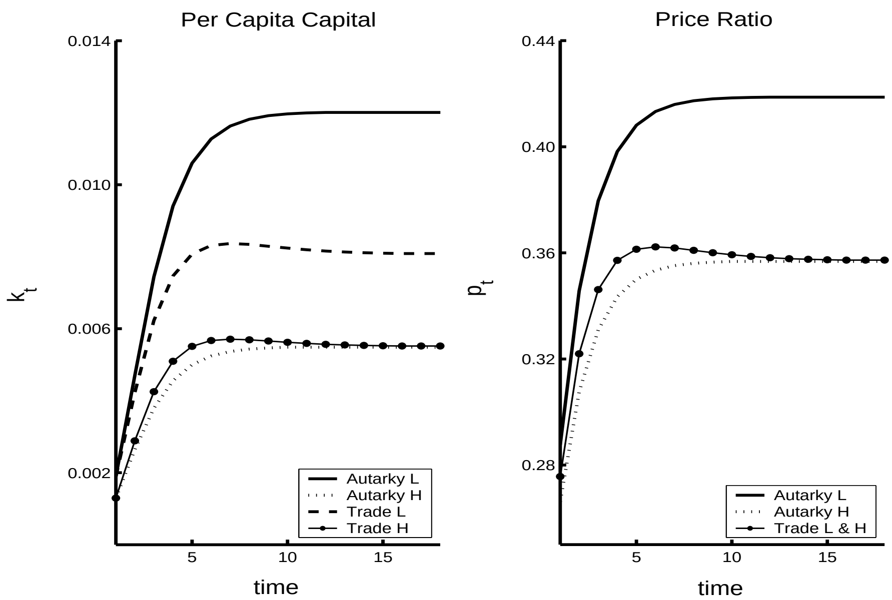

Figure 6.1: Time Paths for Per Capita Capital and the Price Ratio

country L and H results in a common price ratio lying between autarky price ratios. The price ratio path of both countries under free trade is lower than that of the low population growth rate-country under the autarky scenario and higher than that for the high population growth rate-country under the autarky scenario. However, the price ratio path under free trade is closer to that of the high population growth rate-country under autarky than to that of the low population growth rate-country under autarky. The case of China today provides a good example illustrating this point as China is now argued to have become a price setter for labor-intensive manufactured goods in world markets and created a competitive pressure that could trigger a global deflation as noted, for example, by Yang (2003).

The magnitude of the steady state per capita capital and the price ratio under both scenarios are given in Table 6.2. It is clear that, in the long-run free trade results in a substantial decrease in the magnitude of per capita capital of country

L with no change on that of country H, and a considerable decrease in the price ratio of country L with a slight increase in the price ratio of country H.

Table 6.2: Equilibrium Magnitude of k and p under Autarky and Trade

	Autarky		Trade	
	Country L	Country H	Country L	Country H
k_s	0.0120	0.0055	0.081	0.0055
p_s	0.4187	0.3567	0.3573	0.3573

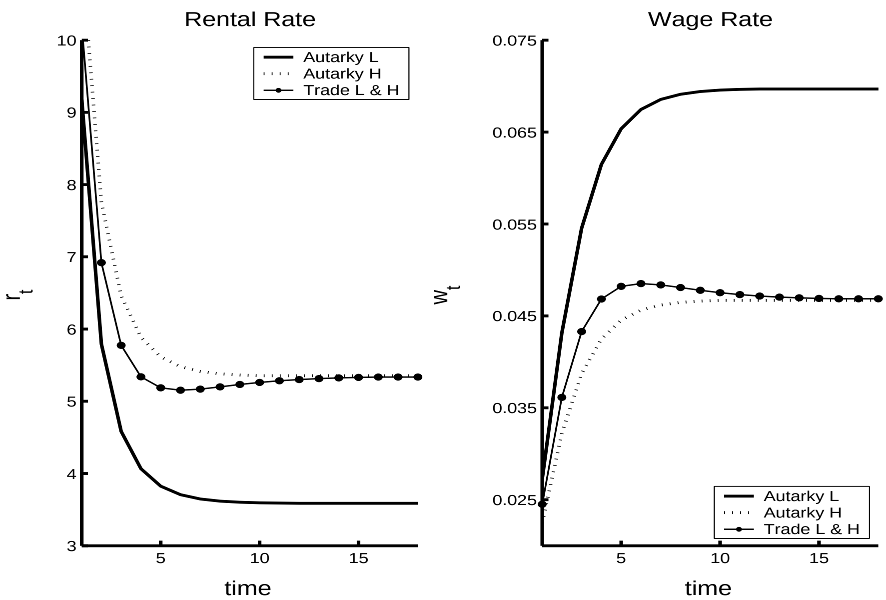

Figure 6.2: Time Paths for the Rental and the Wage Rates

Similar observations can be made for the rental rate and the wage rate. Trade again results in a common rental rate and a common wage rate lying between autarky rates. The time paths of the rental rate and the wage rate are given in Figure 6.2. The common time path of the rental rate under free trade lies between those of the two countries under autarky. However, it lies closer to that of the high population growth rate-country than that of the low population growth rate-country. Hence, in the long-run free trade results in a substantial increase in the rental rate of the low population growth rate-country and a slight decrease in that of the high population growth rate-country. This is also shown by the values of the long-run rental rates of both countries under both scenarios given in Table 6.3.

Table 6.3: Equilibrium Factor Prices under Autarky and Trade

	Autarky		Trade	
	Country L	Country H	Country L	Country H
r_s	3.5880	5.3550	5.3346	5.3346
w_s	0.0697	0.0467	0.0469	0.0469

Similarly, the common time path of the wage rate under free trade lies between those of the two countries under autarky, closer to that of the high population growth rate-country. So, as it is reported in Table 6.3, in the long-run free trade results in a substantial decrease in the rental rate of the low population growth rate-country and a slight increase in that of the high population growth rate-country.

The pattern of trade between these two countries that differ only in the population growth rates is plotted and given in Figure 6.3.

In line with the predictions of the autarky model, the low population growth rate-country L exports commodity 1, the capital-intensive good and imports commodity 2, the labor-intensive good. Whereas the high population growth rate-country H exports commodity 2, the capital-intensive good and imports commodity 1, the labor-intensive good.

Moreover, the long-run non-zero equilibrium values reveal that trade continues in the long-run despite the equalization of prices. This is also observed from the long-run values of the excess supply of each good by each country given in Table 6.4.

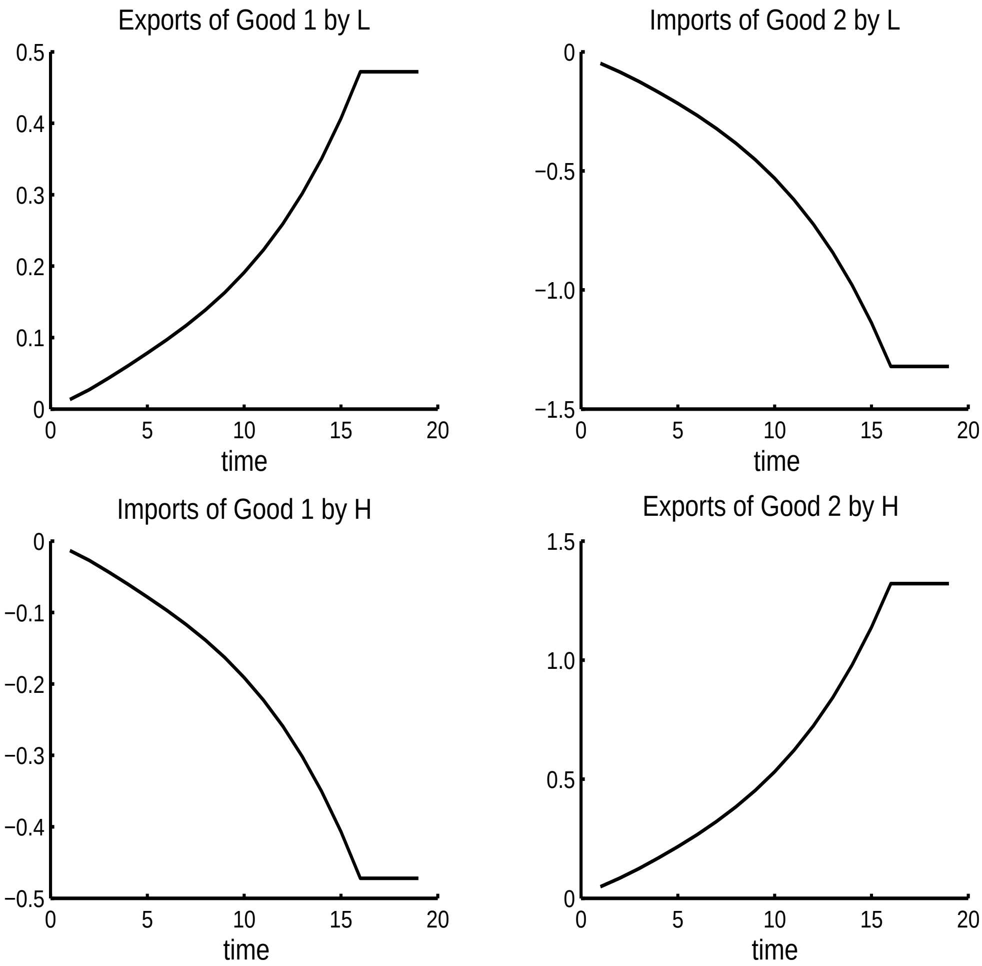

Figure 6.3: Excess Demand for and Supply of Goods

Real per capita consumptions by young of each good in each country under both scenarios are given in Figure 6.4. Free trade results in a considerable decrease in real per capita consumption of good 1 by young in the low population growth rate-country, but just a temporary increase in that of the high population growth rate-country. In a similar way free trade affects real per capita consumption of good 2 by young.

Similarly to the effect of trade on real per capita consumptions of good 1 and

Table 6.4: Excess Demand for Goods by Country

	Country L	Country H
exs_1	0.4721	-0.4721
exs_2	-1.3214	1.3214

where exs_j is excess supply of good j.

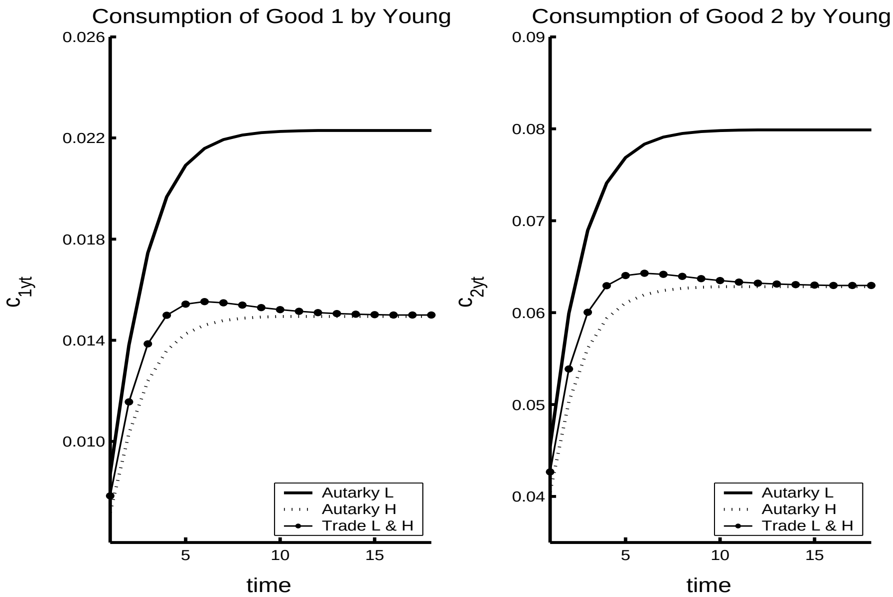

Figure 6.4: Time Paths for the First Period Consumptions

good 2 by young, real per capita consumption of good 1 by old of the low population growth rate-country is decreased and that of the high population growth rate-country is temporarily increased. This is given in Figure 6.5. However, the effect of trade on the long-run real per capita consumption of good 2 by old is different. In the long-run free trade results in a permanent increase in real per capita consumption of good 2 by old in the low population growth rate-country and almost no change in that of the high population growth rate-country.

Hence, in the long-run free trade leads to almost no change in the real per capita consumptions in the high population growth rate-country and a decrease

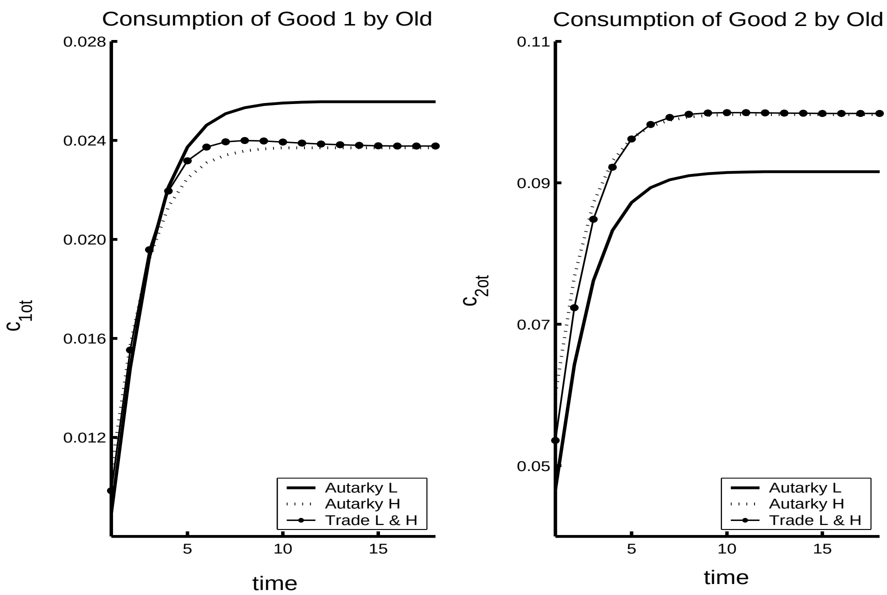

Figure 6.5: Time Paths for the Second Period Consumptions

in the real per capita consumptions in the low population growth rate-country except for the consumption of good 2 by old.

Table 6.5: Per Capita Consumption Equilibrium under Autarky and Trade

	Autarky		Trade	
	Country L	Country H	Country L	Country H
c_{1ys}	0.0223	0.0149	0.0150	0.0150
c_{2ys}	0.0799	0.0628	0.0630	0.0630
c_{1os}	0.0256	0.0237	0.0238	0.0238
c_{2os}	0.0916	0.0996	0.0998	0.0998
u_s	0.0493	0.0387	0.0389	0.0389

From Table 6.5, it is clearly observed that the pre-trade long-run per capita consumption in the low population growth rate-country, country L, is higher than that in the high population growth rate-country, country H, except for the consumption of good 2 by old. Free trade in the long-run makes the high population growth rate-country act as a large country and brings real per capita consumptions in the low population growth rate-country to its autarky real per capita consumption values.

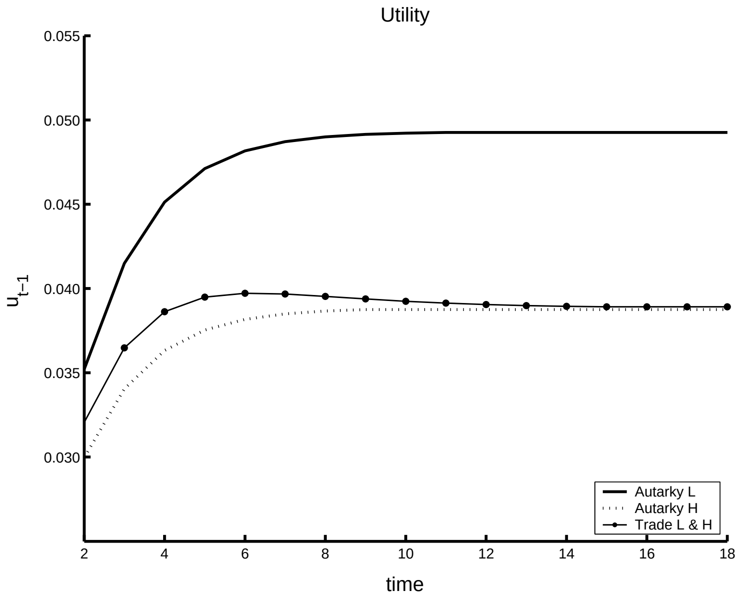

Figure 6.6: Time Paths for the Individual's Utility

Figure 6.6 shows the time paths of utility under trade and autarky scenarios for both countries. Free trade leads to a significant decrease in the utility of the low population growth rate-country and a temporarily increase in that of the high population growth rate-country. Hence, this case clearly shows that not both partners involved into trade gain from trade. In fact the low population growth rate-country would be made worse off by trade, whereas the high population growth rate-country obtains temporary gains that disappear in the long-run.

In this chapter we have analyzed the effect of population growth rate differences on trade between two equal-sized countries. It would also be interesting to look at the case of a small country with a high population growth rate trading with a low population growth country that is initially capable of setting the terms of trade. This scenario involving a low population growth rate-country as an international price setter and the high population growth rate-country as a small country at the time of the opening of trade certainly would be worth studying in the future.

CHAPTER VII

CONCLUSIONS

The foregoing study clearly demonstrates that the long-run closed form solutions of a two-sector, two-generation, two-factor autarky economy are feasible to obtain. As differently from the standard two sector literature, one of the goods is allowed to be used for consumption as well as for investment and the other good is allowed to be used for consumption only. Eventhough this adds to the complexity of the individual's problem, it captures a more realistic set up concerning several consumption goods that can be used for investment purposes as well.

The analysis in chapter 5 has shown that when the model is solved under autarky, differences in the population growth rates alone are observed to give rise to comparative advantages by leading to different relative prices across countries, regardless of initial population sizes of trading countries. In other words, the only difference in demographic characteristics that matters for the direction of product and factor flows is the one between population growth rates. An examination of the sensitivity of the steady state relative price ratio under autarky to changes in population growth rate will indeed identify directions of comparative advantages correctly.

The discussion of the long-run closed form solutions to the 2x2 and 2x2x2 OLG model has shown that of the two countries/regions that are identical in every respect except the population growth rates, the high(low) population growth rate-country will become labor-(capital-) abundant over time, and must be expected to have a comparative advantage in the production of labor-(capital-) intensive commodity, as suggested by the static HO model. Furthermore, we have shown that as long as the population growth rates remain different, there will be room for trade to continue to occur in the long-run. Contrary to the static HO model predictions which states that trade itself would, in the long-run, eliminate the initial differences between relative factor endowments of countries that are assumed to be identical in every other respect, thereby leaving no further incentives for partners to continue trading.

While the population size does not directly affect the long-run equilibrium, the discussion in chapter 6 about the changes that the opening of trade introduced to relative commodity and factor prices prevailing under autarky hinted that initial population sizes of trading nations may play a role in determining the gains from trade over the time path to the steady state. That's because trade empowers the high population growth rate-country H to pull the quantities of all variables towards its own steady state autarky quantities in the long-run. In fact, we have shown that this convergence will be reduced sooner, the larger the difference between population growth rates. This implies that country H will behave as a large country capable of setting the terms of trade in the long-run, as a result of the parallel growth in its share of total world output and population. Symmetrically, the low population growth rate-country L will become a small country, and will begin to act as a price taker in trade. Thus, unless there is a large enough differential in initial population sizes, fast growing population of country H will soon overtake country L in size, and the diverging population sizes will lead to a divergence in the shares of countries in total world output.

If the resulting difference between these shares becomes sufficiently large before the steady state is reached, country H will practically begin to act as the price setter, thereby driving all results. Such a dominance will cause welfare of the world to converge to the autarky welfare of country H, creating welfare losses for country L. It would therefore be correct to argue that initial population size would matter in determining the nature of pre-steady state gains from trade, even though direction of trade itself is determined by differences in population growth rates alone.

Another main contribution is that the analysis in this dissertation has shown that demographically induced differences in relative endowments by themselves are not sufficient for trade to be beneficial to both parties in the long-run, and offered a new explanation for this, adding to previously suggested reasons as to why trade may not be Pareto-superior to autarky in a dynamic, OLG set-up.

SELECT BIBLIOGRAPHY

Auerbach, A. J. and L. J. Kotlikoff. 1987. *Dynamic Fiscal Policy*. Cambridge: Cambridge University Press.

Azariadis, C. 1993. *Intertemporal Macroeconomics*. Cambridge: Blacwell Publishers.

Bardhan, P. K. 1965. "Equilibrium Growth in the International Economy." *Quarterly Journal of Economics* 79:455–464.

Benhabib, J. and K. Nishimura. 1985. "Competitive Equilibrium Cycles." *Journal of Economic Theory* 35:284–306.

Buiter, W. H. 1981. "Time Preferences and International Lending and Borrowing in an Overlapping-Generations Model." *Journal of Political Economy* 89:769–797.

Chen, Z. 1992. "Long-Run Equilibria in a Dynamic Heckscher-Ohlin Model." *Canadian Journal of Economics* 25:923–943.

Chenery, H. B. 1986. Growth and Transformation. In *Industrialization and Growth*, ed. Sherman Robinson Hollis B. Chenery and Moshe Syrquin. Oxford: Oxford University Press.

Deardorff, A. V. 1994. "Growth and International Investment with Diverging Populations." *Oxford Economic Papers* 46:477–491.

Deardorff, A. V. 1999. "Diverging Populations and Endogenous Growth in a Model of Meaningless Trade." *Review of International Economics* 7:359–377.

Diamond, P. A. 1965. “National Debt in a Neoclassical Growth Model.” *American Economic Review* 55:1126–1150.

Eaton, J. 1987. “A Dynamic Specific-Factors Model of International Trade.” *Review of Economic Studies* 54:325–338.

Findlay, R. 1970. “Factor Proportions and Comparative Advantage in the Long Run.” *Journal of Political Economy* 78:27–34.

Fried, J. 1980. “The Intergenerational Distribution of the Gains from Technical Change and from International Trade.” *Canadian Journal of Economics* 13:65–81.

Galor, O. 1986. “Time Preferences and International Labor Migration.” *Journal of Economic Theory* 38:1–20.

Galor, O. 1988*a*. “Is Free Trade Indeed Optimal for a Small Economy? A Dynamic General Equilibrium Analysis.”. Working Paper: 88-08, Brown University.

Galor, O. 1988*b*. “Trade and Welfare in a Small Overlapping Generations Economy.”. Working Paper: 88-04, Brown University.

Galor, O. 1992*a*. “Endogeneous Cycles in a Two-Sector Overlapping-Generations Model.”. Mimeo, Brown University.

Galor, O. 1992*b*. “A Two-Sector Overlapping-Generations Model : A Global Characterisation of the Dynamic System.” *Econometrica* 60:1351–1386.

Galor, O. and A. Mountford. 2003. “Trade, Demographic Transition, and the Great Divergence: Why are a Third of People Indian or Chinese?”. Working Paper: 38-01, Brown University.

Galor, O. and H. E. Ryder. 1989. “Existence, Uniqueness, and Stability of Equilibrium in an Overlapping-Generations Model with Productive Capital.” *Journal of Economic Theory* 49:360–375.

Galor, O. and N. Weil. 2000. “Population, and Growth: From Maltusian Stagnation to the Demographic Transition and Beyond.” *The American Economic Review* 90:806–828.

Galor, O. and S. Lin. 1994. "Terms of Trade and Current Account Dynamics: A Methodological Critique." *International Economic Review* 35:1001–1014.

Galor, O. and S. Lin. 1997. Dynamic Foundations for the Factor Endowment Model of International Trade. In *Dynamics, Economic Growth and International Trade (Studies in International Economics)*, ed. B. S. Jensen and K. Wong. Ann Arbor: University of Michigan Press pp. 151–168.

Guilló, M. D. 2001. "The Trade Balance and the Terms of Trade in a Two-Country Two-Sector OLG Economy." *Spanish Economic Review* 3:71–80.

Kenç, T. and S. Sayan. 2001. "Demographic Shock Transmission from Large to Small Countries: An Overlapping Generations CGE Analysis." *Journal of Policy Modeling* 23:677–702.

Loayza, N., K. Lopez, H. Schmidt-Hebbel and L. Serven. 1998. "Saving in the World: Stylized Facts.". Saving Across the World Project, Wasington DC: The World Bank.

Matsuyama, K. 1988. "Life Cycle Saving and Comparative Advantage in the Long Run." *Economics Letters* 28:375–379.

Matsuyama, K. 2000. "A Ricardian Model with a Continuum of Goods under Nonhomothetic Preferences: Demand Complementarities, Income Distribution, and North-South Trade." *Journal of Political Economy* 108:1093–1120.

Mountford, A. 1998. "Trade, Convergence and Overtaking." *Journal of International Economics* 46:167–182.

Oniki, H. and H. Uzawa. 1965. "Patterns of Trade and Investment in a Dynamic Model of International Trade." *Review of Economic Studies* 32:15–38.

Salvatore, D. 2001. *International Economics.* New York: John Wiley & Sons.

Sayan, S. 2002. "Dynamic Heckscher-Ohlin Results from a 2x2x2x2 Overlapping Generations Model with Unequal Population Growth Rates.". Department of Economics Discussion Paper No. 02-01, Bilkent University, Ankara.

Sayan, S. 2005. "Hecksher-Ohlin Revisited: Implications of Differential Population Dynamics for Trade within an Overlapping Generations Framework.". Forthcoming in Journal of Economic Dynamics and Control.

Sayan, S. and A. E. Uyar. 2001. "Directions of Trade Flows and Labor Movements between High- and Low-Population Growth Countries: An Overlapping Generations General Equilibrium Analysis.". Department of Economics Discussion Paper No. 01-08. Bilkent University, Ankara.

Shell, K. 1967. Optimal Programs of Capital Accumulation for an Economy in which there is Exogenous Technical Change. In *Essays on the Theory of Optimal Economic Growth*, ed. K. Shell. Cambridge MA: MIT Press.

Solow, R. M. 1956. "A Contribution to the Theory of Economic Growth." *Quarterly Journal of Economics* 70:65–94.

Srinivasan, T. N. 1964. "Optimal Savings in a Two-Sector Model of Growth." *Econometrica* 32:358–373.

Stiglitz, J. E. 1970. "Factor Price Equalization in a Dynamic Economy." *Journal of Political Economy* 78:456–488.

Swan, T. W. 1956. "Economic Growth and Capital Accumulation." *Economic Record* 32:334–361.

Tosun, M. S. 2003. "Population Aging and Economic Growth: Political Economy and Open Economy Effects." *Economics Letters* 81:291–296.

Uzawa, H. 1964. "Optimal Growth in a Two-Sector Model of Capital Accumulation." *Review of Economic Studies* 31:1–24.

WDI. 2004. World Bank Group, Data and Statistics. World Development Indicators, Population Dynamics.

Yang, Y. 2003. "China's integration into the World Economy: Implications for Developing Countries.". IMF Working paper: Policy Development and Review Department, WP/03/245.